VOLUME XLV NUMBER 5

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THE ASTROPHYSICAL JOURNAL

AN INTERNATIONAL REVIEW OF SPECTROSCOPY AND ASTRONOMICAL PHYSICS

EDITED BY

GEORGE E. HALE
Mount Wilson Solar Observatory of the Carnegie Institution of Washington

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JUNE 1917

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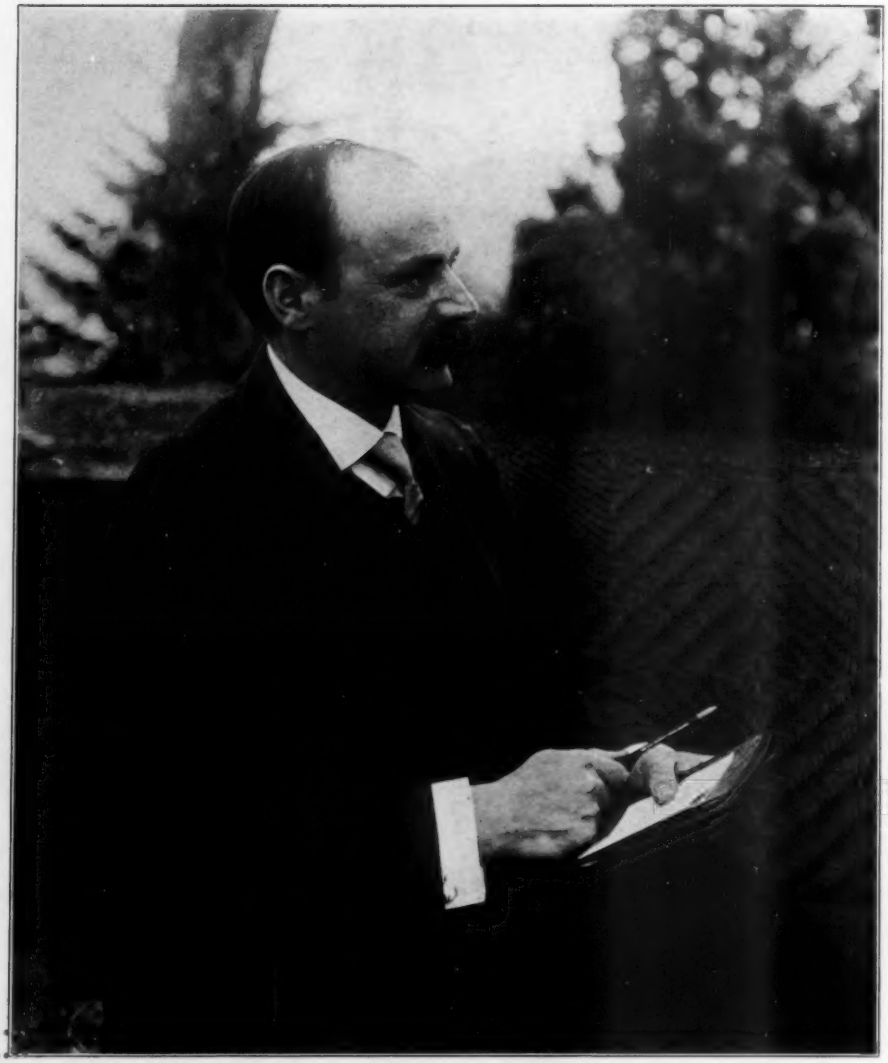

KARL SCHWARZSCHILD

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VOLUME XLV JUNE 1917 NUMBER 5

KARL SCHWARZSCHILD

By EJNAR HERTZSPRUNG

The death of Karl Schwarzschild[1] is a loss to science, all the harder to bear because he was taken away, not "full of years," but when at the summit of productivity.

Not long before his illness Schwarzschild said to the writer: "We have reached the height of life, before long we shall go downward." He did not live to see that time. He died standing. If he had known beforehand that his life would be so short, he could not have used it to better purpose.

Karl Schwarzschild was born at Frankfort-on-the-Main, the son of a Hebrew merchant. Like so many other prominent men, he was the only member of the family who took to science. He manifested his scientific abilities as a schoolboy, at the age of sixteen, when he published two advanced papers on the determination of orbits of double stars, which were printed in 1890 in the *Astronomische Nachrichten*. The next publication, in 1892, contains observations of variable stars. This shows how the direction of Schwarzschild's activity was already fixed in his student years—theory first, practice in the second place. He found his right place at once. "From the beginning," he once said to me, "I have been interested in all branches of astronomy." We may add

[1] Born October 9, 1873; died May 11, 1916.

now: "in all with success." In fact, the many-sidedness of Schwarzschild was astonishing. His high mathematical capacity, so uncommon in the practical astronomer, made him treat the most difficult problems of our science with startling virtuosity from a theoretical point of view. Still it was mathematical sense, not mathematical knowledge, that he most appreciated in others. He had nothing of the inapproachable authority. He was the born teacher and understood how to make the most complicated things clear—so clear that the students were even said to complain that they missed the sensation of dealing with abstruse problems. The readiness with which Schwarzschild grasped the essential point in everything was perhaps his strongest characteristic; the tedious details were left for others of less capacity.

It will be almost impossible for one man to give an adequate sketch of the whole of his scientific labor, because it covered such a wide field. I shall confine myself to some of his papers which are most nearly in the line of this *Journal*, of which he was a collaborator. I have lying before me a complete list of Schwarzschild's papers, carefully selected by Professor Ludendorff. I am sure that Schwarzschild himself would have been astonished to learn that this list contains about 120 numbers.

When we look at the more especially astrophysical work of Schwarzschild, it must be borne in mind that mathematical analysis was what chiefly interested him. An outline of his astrophysical publications alone will therefore cover only the smaller part of his work.

It is characteristic of his papers that they never contain simply bare data, but rather give observations as illustrations for some new method or other item which forms the gist of the communication. Many pearls are hidden in places where we should not naturally look for them. His original paper, of the year 1896, on the measurement of double stars with the aid of a variable objective-grating, in order to diminish the systematic errors of observation, contains also the first measure of an effective wave-length.

Schwarzschild took his degree at Munich in 1896 on a paper treating of Poincaré's theory of rotating liquid bodies. Afterward he was for three years assistant at the von Kuffner Observatory at

Ottakring, near Vienna. It is here that he created the efficient method of photographic stellar photometry with the aid of extra-focal images. In the elaborate paper written on the subject he also gave the well-known formula for a second approximation (the reciprocity law being the first) of the law of density of photographic films, viz., density$=f$ (intensity$\times$exposure-time^q), where the exponent q is a constant. This second approximation has proved to be of good service in the case of long exposure-times used in stellar photometry. Besides photographic magnitudes of a few hundred stars in the Pleiades, Praesepe, and h, χ Persei, the paper contains photographic observations of the two variable stars η Aquilae and β Lyrae. These two last objects both yield a good example of how Schwarzschild's papers never missed the point. The observations of η Aquilae led up to the discovery of the color-variation in the Cepheid variables by showing that the photographic range of variation exceeds the visual range. The photographic observations of β Lyrae, on the other hand, compared with the visual observations of others, yielded a proof of the independence of the velocity of light of the wave-length.

The next two years, from 1899 to 1901, Schwarzschild was active as a privat-docent in the University of Munich.

In the year 1901, at the age of scarcely twenty-eight, he was appointed professor of astronomy and director of the observatory in the University of Göttingen. There he felt at home, not least on account of the especially mathematical sphere of this university. Göttingen he loved above all the other places where he worked. It was also there that he found his wife.

One of the best-known astrophysical works of Schwarzschild during that directorship is the "Göttingen Actinometry," which is to be considered as a continuation of his work on photographic stellar photometry started at Vienna. For the purpose he constructed the *Schraffierkassette*. Instead of using the disk of the intra- or extra-focal star-image for measurement in the microphotometer, the new method effects the necessary broadening of the image mainly through a small and regular shaking of the plate. The main advantage of this device is the considerable increase in the useful part of the field.

The Göttingen Actinometry, completed with this instrument, contains photographic magnitudes of the 3500 stars brighter than 7.5 in the *Bonner Durchmusterung* in the zone 0° to +20° declination. It thus comprises in this region the same stars as the visual Potsdam photometry. The material therefore yields the color-index of every star. Besides this, about 165 stars of the North Polar region were included. A single plate covers 1^h in α and 20° in δ, the focal length being 46 cm. The accuracy reached is considerable, the mean error being about $0^M.03$ for the catalogue magnitude of a star between 5 and 8. It is an interesting fact in connection with the question about the general constancy of the light of the stars that, in spite of the increased accuracy of the single observation, only very few new variables were found or suspected. Among these few the most interesting is the bright variable SZ Tauri, with small range, of the type of ζ Geminorum.

The color-indices of the Göttingen Actinometry showed that the connection between color and spectrum of small dispersion was very intimate—twenty-five cases of considerable discrepancy could practically all be reduced to defective spectral classification. The statistics of color-indices brought to light among the stars between $4^M.5$ and $7^M.0$, visual Potsdam scale, a curious minimum of the number of stars of intermediate color. The fact is intimately related to the phenomenon of "giant" and "dwarf" stars. The predominance of white stars in the Milky Way is also clearly brought out.

In 1905 Schwarzschild went to Algiers to observe the total solar eclipse. Photographs of the flash spectrum were obtained with a spectrograph of "ultra-violet" glass. In the same year he investigated the distribution of ultra-violet light (λ about 320 $\mu\mu$) on the surface of the sun from photographs taken at Jena, using a layer of metallic silver as a color-filter.

Among the several popular papers by Schwarzschild we call attention to a lecture on celestial mechanics, delivered in 1903. It was printed at six different places and deserves special mention for astrophysicists, because it gives an unusually clear outline of theoretical astronomy. In that lecture a sentence occurs which throws a flashlight on his way of teaching. He says: "Poincaré's

work is shrouded in an atmosphere of abstract reasoning which hides his conceptions as the veil which hid the statue at Sais. I have felt the more justified in attempting a simple presentation of Poincaré's work, as this veil has been more rarely lifted than looked upon with respectful awe."

Schwarzschild devoted considerable attention to the construction and the theory of instruments. Among his most valuable contributions on the subject belong his researches on geometrical optics, dating from 1905. He based them on the known theorem of the shortest light-path, the "Eikonal," and developed therefrom the theory of optical instruments. He showed the possibility of constructing a double mirror in the Cassegrain arrangement, which would give a field practically as perfect as that of a photographic refractor of the *carte du ciel* type, in spite of a ratio of aperture to focal length of 1 to 3.5.

Another instrumental invention dating from the time of his residence at Göttingen is the hanging zenith-camera for the determination of the latitude. The instrument yielded as a by-product the declination of 375 stars near the zenith. Furthermore, he constructed a special compass for the navigation of airships.

In 1909 Schwarzschild was appointed director of the Astrophysical Observatory at Potsdam. It was with a heavy heart that he left Göttingen. The new position as the head of an extensive institute claimed much time for non-scientific matter, and it is an evidence of Schwarzschild's indefatigable application that his productivity did not seem to suffer. Not only that; he took the keenest interest in the special work of every member of the staff. His invaluable help was never appealed to in vain; the readiness with which it was given was, for the writer, even a real danger to discretion.

During Schwarzschild's short directorship at Potsdam the most notable instrumental improvement was the regrinding of the two objectives of the great refractor. For the rest, his theoretical and practical work is so many-sided that it is difficult to give a coherent description of it all.

No wonder that the problems of stellar statistics, so accessible to mathematical treatment, especially appealed to his mind. His

best-known achievement in this field is the ellipsoidal hypothesis of stellar movements. As the distribution of stellar velocities showed marked deviations from the Maxwellian law, Kapteyn advanced his hypothesis of two star-streams. But Schwarzschild showed that the unitary consideration would fit the observations nearly as well, if the Maxwellian velocity was only taken different in different directions. In other papers Schwarzschild gave a general solution of the integral equations of stellar statistics. In the continuation of these investigations he obtained among other things the interesting result that the stellar velocity, relative to the sun and projected on the sphere, cannot increase more than to double its amount, for 8 magnitudes decrease in absolute brightness.

In quite another field lie Schwarzschild's investigations relating to the theory of the tails of comets. From the year 1901 dates his well-known theoretical investigation on the pressure of light on small spherical particles. Under the action of the sun such a particle will be attracted proportionally to its volume and repelled by the pressure of light proportionally to its surface. Consequently there will be a determined size of the particle at which these two forces counterbalance each other. On still smaller particles the pressure of light will predominate, and they will hence be repelled from the sun. All this, however, is true only down to a size comparable to the wave-lengths of light, where diffraction begins to play a prominent part and the effect of repulsion is stopped. The maximum ratio between repulsion and attraction is of the order of 20 to 1.

When Halley's comet reappeared in 1910, photographs suitable for photometric use were obtained by the expedition to Teneriffe, equipped by the Potsdam Observatory. This material was discussed by Schwarzschild and showed the important result that the decrease in intensity of the comet's tail with the distance from the head is in the main explained by the corresponding decrease in density. In other words, the luminosity of the repelled particles is retained during their path along the tail, just as would be the case if it were due to resonance of the light of the sun.

At Potsdam, Schwarzschild developed an elaborate theory for the determination of radial velocities with the objective-prism, proposed by E. C. Pickering, and he tried the method practically on the group of the Hyades. In the course of similar work he discovered the strong bright K line in the second-type spectrum of the short-period spectroscopic binary σ Geminorum. The fact may prove to be of special interest in connection with the question about the constitution of the absolutely bright yellow stars.

To the theory of equilibrium of stellar atmospheres, more particularly that of the sun, Schwarzschild has made most valuable contributions. It is natural that this complicated question could be considered only in its main aspects, and his treatment of the subject is a testimony to his intuitive power of recognizing the essential points. In the comparison between the spectrum of the sun—especially the region of the calcium lines H and K—in the center and near the border of the disk he came to the conclusion that the diffusion of light in the sun's atmosphere probably substantially accounts for the observed facts.

Schwarzschild's last paper, written during his illness, bears on the application of the quantum-theory to spectral series. The ardent wish for scientific occupation did not leave him until the last. It seemed incredible to the visitor at his sickbed that he was a man so near death. At such an occasion one feels the mastery of a strong spirit over earthly misery.

Looking backward on his life, we can only preserve in our mind the rich memories he has left behind and try to understand what made him such an unusual man. If I were to name the most characteristic of Schwarzschild's qualities, I would not hesitate to say "elasticity." His mind was always susceptible to new impressions. He had a happy ability for forgetting all petty things—he never brooded over them. It was perhaps the secret of his freshness that he never loaded his mind with immaterial subjects. He was always on the lookout for new ideas. He was in every respect at full development, as if there were nothing more latent in him. Owing to this, he may perhaps not have had much appreciation of dormant capabilities in others—he wanted to see

everything in action as it was in himself. He considered the theoretical mastery over the subject as the highest step on the ladder of the human mind; compared with this, the practical realization was an inferior matter. His judgment about persons and things was quick and sharp and nearly always to the point. At the same time he liked to be contradicted. It was no empty phrase he once used: "I cannot get on with a man who is respectful to me." He wanted "pure" science. A particularly fascinating charm surrounded Schwarzschild. At his loss we feel poorer.

POTSDAM
1917

THE RELATIONSHIP OF STELLAR MOTIONS TO ABSOLUTE MAGNITUDE[1]

By WALTER S. ADAMS and GUSTAF STRÖMBERG

An important product of the radial-velocity observations of recent years is the clear evidence that the stars of large proper motion are characterized by high velocities in space. The fact that large proper motion is to some extent in itself an indication of high velocity must of course enter into consideration, but a computation of space velocities with the aid of mean parallax determinations shows that this cannot account for the results found unless a distribution of velocities is assumed quite other than that of the ordinary law of Maxwell. In a paper published in 1915 Kapteyn and Adams[2] found this result from a discussion of the data for a considerable number of stars, and suggested two alternative hypotheses: (1) that velocity is a function of absolute magnitude; that is, the intrinsically faint stars move more rapidly than the bright stars; (2) that the nearer stars move more rapidly than the distant stars. These conclusions are self-evident, since the stars of large proper motion in general have large parallaxes, and hence are relatively near and intrinsically faint. On the assumption of absolute magnitude as the effective agent, a change of about 1.1 km in radial velocity was found for each unit of magnitude in the case of the K stars.

The results were based wholly upon parallaxes derived from a formula connecting mean parallax with proper motion. The recent development of a method for deriving absolute magnitudes directly from stellar spectra has added greatly to the material available for such an investigation, and we have accordingly taken up the study of the question with a view to discriminating between the suggested explanations. In addition to those stars for which absolute magnitudes have been obtained directly, use has been made of a large

[1] *Contributions from the Mount Wilson Solar Observatory*, No. 131.

[2] *Mount Wilson Communication*, No. 1; *Proceedings of the National Academy of Sciences*, 1, 14, 1915,

number of stars with radial velocities determined at the Lick Observatory. For most of these the parallaxes have been calculated with the aid of a new formula connecting mean parallax with proper motion. The investigation is based upon about 1300 stars of the F, G, K, and M types of spectrum. In the successive portions of this paper we shall endeavor to indicate, first, the evidence afforded by a direct comparison of the arithmetical mean of the radial velocities with absolute magnitudes; secondly, the effect upon the results of the law of distribution of velocities, of stream-motion, and of errors in the parallax determinations; thirdly, the confirmatory evidence afforded by the linear components of the space-velocities.

ARITHMETICAL MEAN OF RADIAL VELOCITIES[1]

As the first and simplest step in the research we have compared radial velocities directly with absolute magnitudes. For this purpose the stars have been divided into groups defined by certain limits of parallax, and within each group the stars have been arranged according to absolute magnitude. The average of the radial velocities freed from the sun's motion is then formed for each mean absolute magnitude within these groups of stars. The results accordingly show the relationship of radial velocity to absolute magnitude among groups of stars located in concentric shells of space around the sun. In this way the effect of magnitude is separated from a possible effect of distance from the sun. In order to secure the large numbers of stars which are necessary for such statistical comparisons, rather wide limits of parallax have been chosen. The results for the F and G stars and for the K and M stars are given separately in Table I. M is the absolute magnitude,[2] and V' the radial velocity corrected for the sun's motion. The number of stars is given in the second column.

A very few stars of quite abnormal velocity have been omitted in the case of the F and G types, but with hardly an exception they are stars of low absolute magnitude, and their inclusion would merely increase the change in V' with absolute magnitude.

[1] In the following pages the arithmetical mean is designated by a bar above the symbol; the geometrical mean, by a bar below.

[2] Absolute magnitude is here defined as apparent magnitude reduced to the distance corresponding to a parallax of 0".1.

The increase in the value of the radial velocity with the absolute magnitude is shown clearly for essentially all of the groups in Table I. It is most marked in the case of the stars of large parallax, since these show the largest range in absolute magnitude, and their parallaxes are most accurately determined. The smallest parallaxes, on the other hand, are subject to much larger percentage errors. The absence of intrinsically faint stars among the groups of

TABLE I

π	Number	$\bar{M}$	$\bar{V}'$	π	Number	$\bar{M}$	$\bar{V}'$
				F and G Stars			
			km				km
$\geq$ 0".050	42	3.0	15.4	0".024 to 0".013	105	0.3	10.8
	57	4.8	16.5		79	1.4	11.7
	29	6.8	21.2		41	3.1	13.5
0.049 to 0.025	56	1.2	15.8	$\leq$ 0.012	83	0.3	11.6
	87	3.0	15.1		58	1.4	14.0
	27	5.6	19.1		19	2.5	15.4
				K and M Stars			
$\geq$ 0.050	29	2.4	20.7	0.024 to 0.017	48	0.6	14.8
	27	6.2	25.1		99	1.4	17.5
	38	7.7	27.2		18	2.7	19.7
	13	10.0	30.2				
					36	0.6	14.3
0.049 to 0.034	36	1.3	21.2	0.016 to 0.013	41	1.3	14.3
	28	2.7	24.3		10	2.2	17.2
	9	6.3	30.7				
					18	0.5	16.3
0.033 to 0.025	23	0.3	13.0	$\leq$ 0.012	33	1.4	15.6
	55	1.5	14.1		13	2.3	13.4
	26	2.4	22.0				

small parallax is, of course, due to the fact that such stars are too faint to have been observed for radial velocity with existing instruments. Interpreted directly, therefore, two general conclusions may be drawn from these results: (1) Among stars at the same distance from the sun, as defined by these parallax limits, there is an increase of radial velocity with decrease in absolute brightness. (2) There is little evidence of a variation in radial velocity with distance from the sun.

If we combine the results for all of the zones according to absolute magnitude we obtain the values given in Table II.

TABLE II

F and G Stars			K and M Stars		
Number	$\bar{M}$	$\bar{V}'$	Number	$\bar{M}$	$\bar{V}'$
		km			km
188	0.3	11.2	125	0.5	14.6
193	1.3	13.6	264	1.4	16.6
189	3.0	14.9	124	2.5	20.6
84	5.1	17.3	74	7.0	26.9
29	6.8	21.2	13	10.0	30.0

A plot of these results in Fig. 1 shows that they are represented with sufficient accuracy by two straight lines nearly, if not quite, parallel to one another. A linear relationship, therefore, is indicated between absolute magnitude and radial velocity, the change

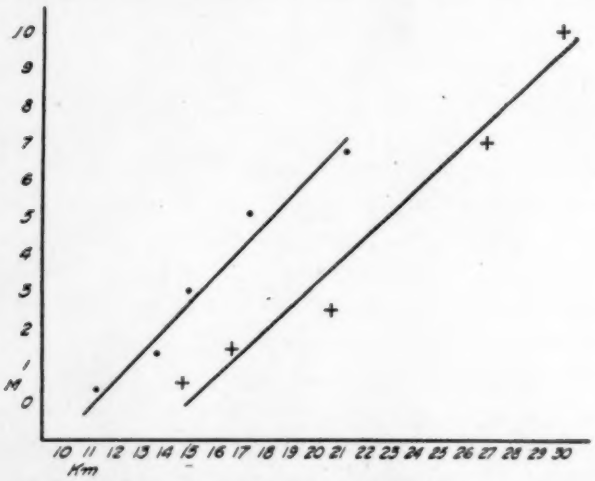


FIG. 1.—Variation of radial velocity with absolute magnitude: points, for F and G stars; crosses, for K and M stars.

in velocity for each unit of magnitude amounting to 1.5 km for the spectral types considered. A value of 1.1 km was found for the stars of type K by Kapteyn and Adams in the paper to which reference has already been made.

DERIVATION OF THE PARALLAXES

The vital importance of accurate parallax determinations for such an investigation is obvious. Since spectroscopic parallaxes were available for only about 700 of the stars used, it became necessary to supplement these values with others derived by aid of a mean parallax formula. A brief description of this formula is therefore essential.

The measurements of van Maanen upon the parallaxes of stars of very small proper motion, together with spectroscopic determinations, have shown that the values for such stars are considerably larger than would be expected from their proper motions. Thus for stars with proper motions less than 0".020 annually the parallaxes appear to be nearly independent of proper motion and dependent only on the magnitude. This is probably due to the fact that a list of stars of very small proper motion, but bright enough to be contained in a catalogue such as that of Boss, involves the selection, not only of distant stars of high intrinsic luminosity, but also of those whose velocity components at right angles to the line of sight are nearly parallel to the sun's velocity component in the same plane and are of the same order of magnitude. Since the intrinsic luminosity of a star cannot be infinitely great, we should expect that the selection of apparently bright stars of very small proper motion would involve a selection of *direction* of space-motion as well as a selection of distant stars. The values given by Kapteyn's mean parallax formula[1] are, therefore, probably too small for stars of very small proper motion.

An additional consideration is that among the distant stars we observe only those of high luminosity. Accordingly, if, as the results of this investigation indicate, the highly luminous stars have small radial velocities and hence small space-velocities, we should expect that the usual mean parallax formula, which is founded on the assumption that the velocity- and the luminosity-curves are independent of distance, would give too small parallaxes for the stars of small proper motion.

For these reasons we have not made use of Kapteyn's equation in its usual form, but have modified it in such a way as to obtain

[1] *Publications of the Astronomical Laboratory of Groningen*, No. 8.

finite parallaxes for zero proper motion by the introduction of an additional constant. The form used is

$$\pi = A(\mu+c)^b \epsilon^m \text{ or } \log \pi = \log A + b \log (\mu+c) + m \log \epsilon.$$

The spectroscopically observed parallaxes were used in the derivation of the constants in order to render the results homogeneous. Since the value of b was found nearly equal to 1, and it proved most difficult to determine b and c separately, the more simple form

$$\log \pi = \log A + \log (\mu+c) + m \log \epsilon$$

was used for stars with proper motions less than 1″, and gave results in good agreement with observation. Since proper motion is a different measure of parallax in the neighborhood of the sun's apices and apart from them, the constants have been determined separately for the stars near and distant from these points. The formula has been referred to the magnitude scale of the Harvard system.

The mean errors in $\log \pi$ derived from this equation are dependent upon the magnitude and proper motion. They are largest for the stars of types K and M, but are considerably smaller than those given by the formulae of Kapteyn and van Rhijn.[1] Further details of this formula and its use will be given in a future communication.

FREQUENCY DISTRIBUTION OF VELOCITIES

Reference has already been made to the suggestion in the paper by Kapteyn and Adams that the distribution of velocities in space might be quite other than that given by Maxwell's law, and that this fact might be used to explain the smaller radial velocities of the stars of small proper motion. It is evident that, if in any selected group of stars there is an excess of large velocities as compared with that given by Maxwell's law, we are inclined to assign such stars too large parallaxes owing to their large proper motions, and consequently to conclude that the mean intrinsic brightness of the stars in the group is too low. We might, therefore, arrive at false conclusions regarding a relationship between velocity and

[1] *Mt. Wilson Contr.*, No. 110; *Astrophysical Journal*, 43, 36, 1916.

either distance or absolute magnitude. It is therefore most desirable that the effect of frequency distribution be eliminated so far as possible. The following method was devised by Strömberg for computing the parallaxes in a manner independent of the law of velocity-distribution.

The star's velocity in space is projected upon three axes, one of which corresponds to radial velocity, and the other two to the components of proper motion which are denoted by ν and τ in Kapteyn's notation. The ν axis is directed toward the sun's apex, and the τ axis is at right angles to the ν axis. The three components of the star's velocity in space, corrected for the sun's motion, are as follows:

$$V' = V + V_0 \cos d; \qquad V_\nu = \frac{K\nu}{\pi} + V_0 \sin d; \qquad V_\tau = \frac{K\tau}{\pi},$$

in which V is the star's radial velocity, V_0 the sun's velocity, d the angle between the star and the sun's apex, K a constant[1] equal to 4.738, and ν and τ the components of the proper motion. If we assume that the frequency function F for these three components is the same, without any assumption as to its nature, we have

$$F(V') = F(V_\tau) \text{ or } F_1(\log V') = F_1(\log V_\tau) \text{ or } \overline{\log V'} = \overline{\log V_\tau}.$$

Denoting mean values by a bar above the symbol, we find

$$\overline{\log \pi} = \log K + \overline{\log \tau} - \overline{\log V'}. \tag{1}$$

If the *geometrical* means, denoted by a bar below the symbol, are then introduced, we have

$$\log \underline{\pi} = \overline{\log \pi}; \qquad \log \underline{\tau} = \overline{\log \tau}; \qquad \log \underline{V'} = \overline{\log V'}$$

or, finally,

$$\underline{\pi} = K \frac{\underline{\tau}}{\underline{V'}}. \tag{2}$$

This equation is of the same form as that derived by Campbell for the arithmetical means and valid for a small range in the parallaxes. Equation (2), however, is correct whatever the variation in π, τ, or V'. In equations (1) and (2) the numerical values of τ and V' are to be considered without regard to sign.

[1] W. W. Campbell, *Stellar Motions*, p. 222.

A similar computation using the ν component gives as a final result

$$\bar{\pi} = \frac{K\nu}{(V' - V_0 \sin \bar{d})}. \qquad (3)$$

The two equations (2) and (3) have been used to compute the parallaxes for the groups of stars used in Table I. These results are shown in Table III, as well as the geometrical means $\bar{\pi}$ of the parallaxes employed.

TABLE III

Number	π	$\bar{\pi}$	$\bar{\pi}(\tau)$	$\bar{\pi}(\nu)$
	F AND G STARS			
142	≷0".050	0".082	0".078	0".086
95	0.049–0.034	.041	.045	.044
91	.033– .025	.028	.023	.021
122	.024– .017	.020	.017	.013
110	.016– .013	.015	.006	.004
68	0.012– .010	.011	.004	.003
97	≷0.009	0.007	0.003	0.003
	K AND M STARS			
108	≷0".050	0".094	0".075	0".103
73	0.049– .034	.039	.021	.030
103	.033– .025	.028	.019	.022
162	.024– .017	.020	.010	.010
86	0.016– .013	.015	.004	.004
63	≷0.012	0.009	0.003	0.004

This comparison shows a good degree of accordance for the groups of large parallax between the values of the parallax actually employed and the computed values from the ν and τ components. For the groups of small parallax, however, the values used are appreciably larger than the computed ones. A part of this difference is due to the selection of stars with motions mainly in the line of sight and tangential components parallel and nearly equal to the sun's motion. This is seen from the results of Table IV where the radial velocities for the distant stars are considerably larger than the linear τ components. It is probable, however, that the parallax values employed are subject to a certain amount of systematic

TABLE IV

π	M	$\bar{M}$	$\bar{\pi}$	$\underline{V'}$	$\underline{V}_\pi$	$\frac{1}{2}(\underline{V'}+\underline{V}_\pi)$	No.	$\bar{V'}$	No.
				km	km	km		km	
F and G Stars									
≧0".050	≦3.9..	2.9	0".076	12.7	8.8	10.8	41	15.3	42
	4.0–4.9..	4.47	.082	13.4	15.2	14.3	35	16.8	34
	5.0–5.9..	5.35	.080	16.9	16.5	16.7	33	16.0	23
	≧6.0..	6.92	.094	19.2	16.9	18.0	33	21.2	29
0.049–0.034	≦1.9..	1.28	.040	9.3	7.9	8.6	19	13.9	19
	2.0–2.9..	2.51	.039	14.6	9.6	12.1	24	15.0	19
	3.0–4.9..	3.73	.042	12.9	21.1	17.0	26	15.4	24
	≧5.0..	5.62	.042	23.2	36.9	30.0	26	20.0	21
0.033–0.025	≦1.9..	1.09	.029	10.5	7.0	8.8	37	16.9	37
	2.0–2.9..	2.32	.028	9.7	7.3	8.5	31	14.6	31
	≧3.0..	4.61	.029	19.0	26.2	22.6	23	15.9	19
0.024–0.017	≦0.9..	0.24	.020	6.2	2.9	4.6	43	8.6	42
	1.0–1.9..	1.44	.020	7.6	8.6	8.1	50	13.6	49
	≧2.0..	3.37	.020	11.5	16.8	14.1	29	11.9	25
0.016–0.013	≦≧1.9..	0.73	.015	7.6	3.4	5.5	94	12.7	93
	≧2.0..	3.10	.015	11.3	4.6	7.5	16	15.9	16
0.012–0.010	≦≧0.9..	0.37	.011	7.1	3.4	5.3	36	11.4	35
	≧1.0..	1.52	.011	11.0	4.0	7.5	32	14.1	31
≦0.009	≦≧0.9..	0.21	.007	7.8	3.4	5.6	49	11.7	48
	≧1.0..	1.73	0.007	14.0	6.7	10.4	48	14.5	46
K and M Stars									
≧0".050	≦2.9..	1.95	0".066	11.1	8.2	9.6	19	17.2	19
	3.0–6.9..	5.38	.081	14.8	13.2	14.0	38	25.7	37
	7.0–7.9..	7.27	.092	18.8	18.1	18.5	24	27.5	24
	≧8.0..	9.34	.149	22.2	23.1	22.6	27	28.4	27
0.049–0.034	≦1.9..	1.29	.038	16.9	6.2	11.6	36	21.2	36
	2.0–3.9..	2.69	.040	18.1	12.1	15.1	27	24.3	28
	≧4.0..	6.06	.041	24.3	23.1	23.7	9	30.7	9
0.033–0.025	≦0.9..	0.26	.028	6.8	3.1	5.0	23	13.0	23
	1.0–1.9..	1.49	.028	8.2	7.0	7.6	55	14.1	55
	≧2.0..	2.46	.029	17.8	9.4	13.6	25	22.0	26
0.024–0.017	≦0.9..	0.59	.020	9.8	3.3	6.6	46	14.8	48
	1.0–1.9..	1.40	.020	12.2	6.7	9.5	98	17.5	99
	≧2.0..	2.68	.021	14.7	8.3	11.5	18	19.7	18
0.016–0.013	≦≧0.9..	0.61	.015	10.0	2.5	6.2	35	14.5	36
	≧1.0..	1.48	.015	10.4	2.9	6.7	51	14.8	51
≦0.012	≦≧0.9..	0.53	.008	9.1	3.4	6.3	18	16.3	18
	1.0–1.9..	1.39	.009	11.8	4.9	8.4	32	15.6	33
	≧2.0..	2.32	0.009	11.3	.....	....	13	13.4	13

error, based as they are in the final analysis upon an extremely scanty amount of observational material.

EFFECT OF STREAM-MOTION

The stream-motion of the stars used in this investigation might affect to some extent the means of the radial velocities, although with the relatively large groups employed the influence would certainly be slight. As a means of reducing the effect, we have computed the geometrical means of the radial velocities $\underline{V'}$ and the linear τ components V_τ, and have formed the arithmetical means of these two values. If stream-motion is to be regarded as affecting mainly the direction of the space-velocities, the influence upon the mean of the two components should be greatly diminished. Table IV contains these results. The linear τ component $\underline{V_\tau}$ is calculated from the geometrical mean of the parallaxes of the stars in the group by the aid of the formula

$$\log \underline{V_\tau} = \log K + \overline{\log \tau} - \overline{\log \pi}.$$

For purposes of comparison the arithmetical mean $\overline{V'}$ is added for the different magnitude groups.

The results of Table IV agree with those of Table I in indicating a decided increase in velocity with decreasing absolute brightness among the stars of the several zones. This is true for both of the components used. There seems to be little evidence of a decrease in velocity with distance for a constant absolute magnitude, and it appears certain that the effect if present must be relatively slight.

As a final test of the results, we have used only the stars with parallaxes exceeding $0''.017$. For these we have computed the geometrical means of the radial velocities (excluding a few stars with velocities smaller than 0.3 km), and the corresponding τ and ν parallaxes; also the absolute magnitudes and the linear τ components, using the means of $\underline{\pi}(\tau)$ and $\underline{\pi}(\nu)$. The absolute magnitudes and the τ components, therefore, are independent of measured parallaxes and of the frequency-distribution of the velocities. A single exception is found in the last group of the K and M stars for which the observed parallaxes are used. The results are given in Table V.

The increase of velocity with decrease of absolute brightness is shown very clearly in these results. The same result is true for the apparent brightness $\overline{m}$ of Table V. Under this heading are given the arithmetical means of the apparent magnitudes of the stars included within the groups M.

TABLE V

M	No.	$\pi(\tau)$	$\pi(v)$	$\frac{1}{2}(\pi(\tau)+\pi(v))$	$\overline{m}$	$\overline{M}$	V'	V_τ	$\frac{1}{2}(V'+V_\tau)$
F and G Stars									
							km	km	km
≦0.9....	60	0".012	0".010	0".011	3.38	−1.43	6.33	6.69	6.51
1.0–1.9....	94	.023	.018	.020	4.44	+0.99	9.56	10.57	10.06
2.0–2.9....	78	.024	.026	.025	4.84	1.83	10.86	10.59	10.73
3.0–3.9....	52	.094	.073	.084	4.77	4.38	10.31	11.64	10.97
4.0–4.9....	62	.060	.059	.060	5.84	4.71	15.97	16.01	15.99
5.0–5.9....	64	.073	.057	.065	6.69	5.74	19.54	21.97	20.75
≧6.0....	40	0.077	0.095	0.086	7.32	6.99	20.96	18.78	19.98
	450	0.036	0.034	0.035	5.18	2.89	11.89	12.28	12.08
K and M Stars									
≦0.9....	77	0.009	0.007	0.008	3.54	−1.89	8.54	9.29	8.92
1.0–1.9....	188	.015	.018	.017	4.44	+0.54	11.65	10.42	11.04
2.0–2.9....	66	.022	.026	.024	4.64	1.57	14.23	13.14	13.69
3.0–5.9....	39	.035	.060	.048	5.55	3.95	20.59	15.20	17.90
6.0–7.9....	48	.081	.124	.102	7.33	7.38	17.97	14.17	16.07
≧8.0....	29	(0.137)	(0.203)	0.147	8.42	9.25	17.88	16.61	17.25
	447	0.022	0.026	0.024	4.98	1.88	12.88	11.64	12.26

RADIAL VELOCITY AND SPECTRAL TYPE

A comparison of the average radial velocity of the F and G stars with that of the K and M stars is complicated by the fact that the former show a much greater velocity-dispersion. Two stars with velocities of over 300 km belong to the F and G group, and there are several others of quite abnormal velocity. These stars have been excluded in the absolute-magnitude discussion, and it is for this reason that Table II cannot be regarded as giving the direct relationship between spectral type and velocity. For these reasons it seems probable to us that the use of the geometrical mean, which is not abnormally influenced by the inclusion of stars of

exceptionally high velocity, is preferable for this purpose. Table VI gives the geometrical means of the radial and component velocities together with one-half their sum. The parallaxes have been corrected for systematic errors. All stars with parallaxes exceeding 0".017 have been included.

TABLE VI

No.	M	$\pi(r)$	$\pi(v)$	π	$\bar{M}$	$\underline{V}'$	$\underline{V}_r$	$\frac{1}{2}(\underline{V}'+\underline{V}_r)$
				F AND G STARS				
						km	km	km
154....	$\leqq$ 1.9	0".017	0".014	0".016	+0.01	8.14	9.00	8.57
301....	$\geqq$ 2.0	0.046	0.044	0.045	+4.03	13.89	14.07	13.98
				K AND M STARS				
268....	$\leqq$ 1.9	0.013	0.014	0.014	−0.13	10.31	9.91	10.11
184....	$\geqq$ 2.0	0.043	0.056	0.050	+4.62	16.17	14.15	15.16

These results indicate that the K and M type stars have a velocity of from 1.0 to 1.5 km higher than the F and G stars of the same absolute magnitude.

CONCLUSIONS AND SUMMARY

1. Radial velocity is a function of absolute magnitude for the 1300 stars used in this investigation. The increase in velocity is about 1.5 km for a decrease in brightness of one magnitude.

a) This effect cannot be ascribed to distance from the sun, since stars at the same distance show the effect strongly.

b) It cannot be due to the law of frequency-distribution of the velocities, since the component velocities calculated under no assumption of the nature of the frequency-law show the effect equally with the radial velocity.

c) It cannot be due to the effect of stream-motion.

2. A modified form of the formula connecting mean parallax with proper motion has been derived and applied to the stars under investigation. This formula gives mean errors considerably below those of the formulae commonly in use.

3. A method has been devised for calculating the geometrical means of the parallaxes of groups of stars, and these values have

been compared with those used in the investigation. The agreement is good for the groups of large parallax; for those of smaller parallax the observed values are somewhat too large.

4. The radial velocity and the τ-component velocity increase with decrease in apparent brightness.

5. The giant and dwarf division among the K and M stars is indicated clearly in the absolute magnitudes (Table V).

6. The use of the geometrical mean of the velocities and the parallaxes possesses some marked advantages for an investigation of this character. In the case of velocities it is unnecessary to exclude abnormally high values, as was done for the arithmetical mean, but only values very near zero. The agreement of the two means in indicating the variation of velocity with absolute magnitude is, therefore, a valuable check on the results. Further, from the geometrical mean of the parallaxes, it is possible to compute directly the mean absolute magnitude by the formula $\overline{M}=\overline{m}+5+5 \log \underline{\pi}$, whatever the range in m and π.

7. The stars of types K and M have mean velocities about 1.0 or 1.5 km higher than the F and G stars of the same absolute magnitude. This conclusion is based upon a discussion of the geometrical means of the radial and component velocities.

MOUNT WILSON SOLAR OBSERVATORY
April 1917

COMPARISON OF VISUAL AND PHOTOGRAPHIC OBSERVATIONS OF ECLIPSING VARIABLES

By HENRY NORRIS RUSSELL, MARY FOWLER, AND MARTHA C. BORTON

The present paper contains the first results of a program of co-operation between the Harvard and Princeton observatories in the measurement and discussion of the observations of variable stars contained in the plates of the Harvard Photographic Library.

The photographic magnitudes have been estimated at Harvard by comparison with a sequence of stars in each field for which magnitudes were derived by Miss Leavitt by comparison with the Polar Sequence. For the stars W Delphini, S Cancri, and SW Cygni the estimates were made in 1903, as part of the work done under a grant from the Carnegie Institution; but the magnitudes of the comparison stars have been redetermined in 1916 by Miss Leavitt. For the remaining stars the estimates were made in 1915 and 1916 by Miss Leavitt and Miss Locke.

The discussion of the observations has been carried on at the Princeton University Observatory, this part of the work having been made possible by a gift to the Observatory by Mr. Archibald D. Russell. For the stars here discussed the computations have been made and the published accounts prepared by Miss Fowler in the case of W Delphini, U Sagittae, and S Cancri; by Miss Borton for RW Tauri and SW Cygni; and by both together for W Ursae Majoris. The observations upon which these reductions are based will be published in the *Harvard Annals*, together with the final phase for each and residuals (O−C) from the light-curves adopted below.

The concluding section, containing a general discussion of the work so far completed, has been written by Professor H. N. Russell.

1. W DELPHINI ($20^h33^m.1, +17°56'$ [1900])

When Wendell's later observations, given in *Harvard Annals*, **69**, Part II, which have been published since Professor Russell computed the light-curve based on Wendell's earlier observations, were

being plotted, suspicion of a changing period was aroused and all Wendell's observations were studied further. They were divided into seven groups, with the result given in Table I. The fourth column gives the correction (O−C) to Wendell's elements of minimum: 2,412,002.5971+4.8061 E. These results were plotted and it was found that a symmetrical curve, whose equation gives the correction $-0^d0377+0.000{,}111\ E-0.000{,}000{,}062\ E^2$, could be drawn through them. After this is applied, the outstanding residuals are those given in the column headed O−C′.

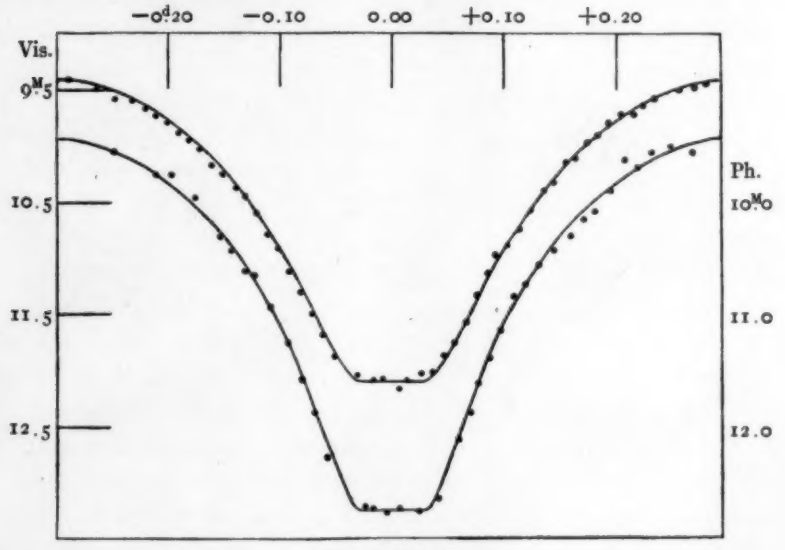
TABLE I

No.	Mean Epoch	No. of Obs.	O–C	O–C′
		WENDELL'S OBSERVATIONS		
1.....	+ 322	20	-0^d0125	-0^d0035
2.....	+ 381	128	− .004	+ .001
3.....	+ 458	146	.000	.000
4.....	+ 695	15	+ .010	+ .0005
5.....	+ 772	25	+ .010	− .001
6.....	+1222	14	+ .006	.000
7.....	+1374	14	−0.002	0.000
		PHOTOGRAPHIC OBSERVATIONS		
1.....	+ 360	113	−0.006	0.000
2.....	+ 482	166	+0.002	0.000
3.....	+ 634	71	+0.009	+0.001
4.....	+ 810	13	+0.012	0.000

Phases for the 401 photographic observations were computed first with Wendell's elements and then corrected to the smooth curve given above. They were then divided into four groups in order of phase, plotted, and the corrections to Wendell's elements obtained, as shown in the lower part of Table I, which fully confirm the correction to Wendell's period during the interval which they cover.

Wendell's elements, thus corrected, which are $2412002.5594+4.806211\ E-0.000{,}000{,}062\ E^2$, are those used in computing the light-curve here discussed. Wendell's elements, as given in *Harvard Annals*, **69**, however, are probably of more value for purposes of prediction, owing to the probability that the effect here appearing as the term in E^2 may ultimately be found to be periodic in nature.

In applying this correction to the visual observations, Wendell's earlier observations on the ascending branch and on the descending branch were treated separately. The mean epoch of 149 observations on the ascending branch is 423.6 and that of 204 observations on the descending branch is 453.3. The corrections for these two epochs are $+0^d.0008$ and $+0^d.0001$, respectively; but a further allowance must be made for the quadratic term, $+0.000,000,062\ E^2$.



W Delphini
Visual, above; photographic, below

FIG. 1

The mean square difference in epoch between an individual observation and the mean epoch of the group is ±111 for the ascending and ±114 for the descending branch. By applying the value of the quadratic term for these intervals, the complete corrections are found to be $+0^d.0016$ and $+0^d.0008$, respectively, which fortunately are so nearly equal that it will suffice to apply the mean, $+0^d.0012$, to the computed epoch without changing our light-curve or elements. The epoch assumed by Professor Russell in his determination of the light-curve in *Astrophysical Journal*, **36**, 133, 1912, is $0^d.0015$ earlier than Wendell's and hence $0^d.0027$ earlier than the new elements.

The 390 photographic observations which remained after rejecting eleven which had inadmissably large residuals were combined

TABLE II

NORMAL MAGNITUDES OF W DELPHINI

Phase	Mag.	No. of Obs.	O−C
+0^{d}0062.....	12.738	5	−0^{M}01
.0235.....	12.755	2	+ .01
.0417.....	12.643	10	+ .13
.0597.....	12.111	10	+ .02
.0702.....	11.875	11	+ .08
.0776.....	11.614	11	.00
.0871.....	11.398	11	+ .01
.0972.....	11.161	10	+ .02
.1088.....	10.842	10	− .09
.1183.....	10.748	9	− .02
.1312.....	10.560	10	.00
.1433.....	10.428	10	+ .04
.1583.....	10.303	9	+ .08
.1707.....	10.153	7	+ .08
.1807.....	10.078	9	+ .08
.1953.....	9.899	10	+ .02
.2067.....	9.620	9	− .14
.2186.....	9.680	8	− .03
.2314.....	9.569	12	− .05
.2495.....	9.506	12	− .05
+ .2684.....	9.552	9	+ .08
− .2484.....	9.544	9	− .02
.2105.....	9.740	10	.00
.1959.....	9.752	9	− .07
.1758.....	9.964	10	− .06
.1534.....	10.302	11	+ .04
.1428.....	10.427	12	+ .06
.1313.....	10.621	11	+ .11
.1225.....	10.650	11	− .01
.1079.....	10.945	12	.00
.0925.....	11.271	11	+ .04
.0799.....	11.580	12	+ .04
.0689.....	11.875	11	+ .01
.0506.....	12.278	12	+ .11
.0243.....	12.720	6	− .03
.0174.....	12.738	5	− .01
− .0046.....	12.774	5	+0.02
0.302.....	9.46	6	+0.03
0.349.....	9.31	5	− .12
2.67.....	9.30	3	− .13
3.32.....	9.45	6	+ .02
3.44.....	9.37	5	− .06
3.67.....	9.35	5	− .08
4.459.....	9.48	7	+0.05

into thirty-seven normals, which are given in Table II. Constant minimum light determined from twenty-three observations was

found to be 12.743, and constant maximum light from thirty-nine observations to be 9.377, or 9.405 if two very discordant observations were rejected. The visual light-curve, given by Professor Russell,[1] was then readjusted to the photographic relation between the relative brightness of the two stars by multiplying the visual loss of intensity from maximum light $(1-l)$ by 1.0417. This curve was then applied to the photographic normals and was found to be too high throughout the whole upper part of the curve. As maximum light was determined from so few observations in comparison with the large number of observations on the upper part of the curve, all of which had positive residuals, it seemed best to take the latter into consideration, and maximum light was therefore assumed to be 9.43, and minimum light 12.75. This shows a range of light of $3^{M}.32$ or a loss of light of 0.9530, while the visual depth is only $2^{M}.70$, the range being from 9.40 to 12.10, or a loss of light of 0.9168 of the whole.

The computed curve being readjusted to this new relation by changing the factor from 1.0417 to 1.0395, the light-curve given in Table III was adopted as satisfactory as regards the range of

TABLE III

t	Visual Mag.	Visual Δm	Visual $1-l$	Photo. $1-l$	Photo. Δm	Photo. Mag.
$0^{d}.030$	12.10	2.70	0.9168	0.9530	3.32	12.75
.040	12.01	2.61	.9096	.9455	3.16	12.59
.050	11.86	2.46	.8962	.9316	2.91	12.34
.060	11.68	2.28	.8775	.9122	2.64	12.07
.070	11.50	2.10	.8555	.8893	2.39	11.82
.090	11.09	1.69	.7891	.8203	1.86	11.29
.110	10.77	1.37	.7169	.7452	1.48	10.91
.140	10.33	0.93	.5754	.5981	0.99	10.42
.160	10.10	0.70	.4752	.4940	0.74	10.17
.180	9.94	0.54	.3919	.4074	0.57	10.00
.200	9.78	0.38	.2953	.3070	0.40	9.83
.220	9.64	0.24	.1983	.2061	0.25	9.68
.240	9.54	0.14	.1210	.1253	0.15	9.58
.260	9.48	0.08	.0710	.0738	0.08	9.51
0.300	9.40	0.00	0.0000	0.0000	0.00	9.43

In this table Δm denotes the difference in stellar magnitude between the brightness of the star at any given moment and at full light, and $1-l$ the loss of light at this moment, in terms of the whole light of the system as a unit.

variation and form of the curve. A least-square solution for the magnitude at maximum and the epoch of minimum was then made,

[1] *Astrophysical Journal*, 36, 134, 1912.

the conditional equations being of the form $x+sy=n$, where s is the slope of curve at the phase of a given normal, n the residual O−C for this normal from the provisional curve, x a correction to the assumed magnitude at maximum, and y a correction to the assumed epoch of mid-eclipse. In the present case, x and n were expressed in hundredths of a magnitude, and y in thousandths of a day, so that $s=1$ corresponded to a rate of variation of a magnitude in $0^{d}.10$.

The values derived from thirty-two conditional equations were $x=+0^{M}.0169\pm0^{M}.0077$ and $y=-0^{d}.00016\pm0.00046$, that is, the epoch of minimum derived from the photographs is earlier by 0.23 ± 0.66 minutes than that given by the formula previously adopted.

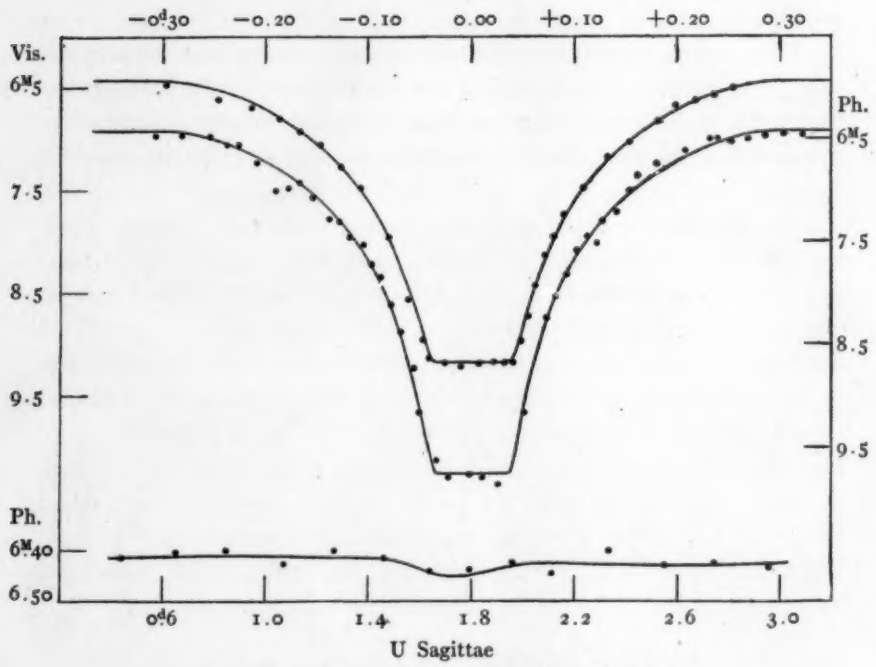
The probable error of one normal magnitude was $\pm0^{M}.043$. As the average number of observations combined into a normal was 9.5, this corresponds to a probable error of $\pm0^{M}.132$ for a single photographic observation.

A least-square solution was not made for the visual observations on account of the uncertainty concerning the nature of the variable term in the expression for the period; but the value found above—according to which the visual minimum comes $0^{d}.0027$, or 3.9 minutes, earlier than indicated by the formula—must be very near the truth. The photographic minimum therefore comes $3^{M}.7$ later than the visual—a difference decidedly too great to be explained by any uncertainty in the variable correction aforesaid.

2. U SAGITTAE ($19^{h}14^{m}.4,+19°26'$)

Five hundred and eighty photographic observations of U Sagittae were available, twenty of which were eventually rejected because marked uncertain by the observers, and nine on account of large and unexplained discordances. These observations were made twice on the majority of plates, first by Miss Locke and then by Miss Leavitt, at Cambridge. The instruments used were the 1-inch Cooke Anastigmat in Cambridge, the 1-inch Cooke in Arequipa, and the 0.5-inch Ross-Zeiss in Cambridge. The exposures were usually 60 minutes. Some plates with the 1-inch Cooke were taken with several successive exposures, usually of ten minutes each, but sometimes of 30 minutes. Owing to these long exposures, the brightness

recorded on the photograph, when the star is varying rapidly, will not be exactly that corresponding to the moment of mid-exposure. The necessary correction can easily be determined from a provisional light-curve, if it is assumed that the photographic action is proportional to the intensity of the star's light, and is simply the difference between the mean of the intensity during the exposure and the



U Sagittae

Upper curve, visual; two lower curves, photographic

FIG. 2

intensity at the middle of exposure. This correction is greatest when half the exposure falls in the constant phase at minimum and half in the rapid increase or decrease, and may amount to $0^{M}.21$ for a one-hour exposure or $0^{M}.05$ for a half-hour exposure, but is usually insensible. The long exposures in this case nearly all occur during constant maximum light, so that corrections for long exposure were applied in only 20 cases, and many of these are for half-hour exposures and therefore are small.

Phases were first computed with Graff's[1] elements: minimum = 2,417,130.4133+3.380,604 E. Wendell's 92 early and 36 later observations were reduced with these elements. On plotting the photographic times of minimum, it was quite evident that the period needed revision, and, taking into consideration Wendell's, Graff's, Maggini's,[2] and the Harvard photographic observations, we find that the best period seemed to be $3^d.380621$, with no definite indication of a variable period.

Forty-five normals were then formed from the 202 observations during eclipse, with phases corrected to this new period and magnitudes corrected for length of exposure. Wendell's visual observations were also collected into 32 normal groups. Tables IV and V

TABLE IV

NORMAL MAGNITUDES OF U SAGITTAE—HARVARD PHOTOGRAPHS

Phase	Mag.	No. of Obs.	O−C	Phase	Mag.	No. of Obs.	O−C
+0.0109..	9.793	7	+0.04	−0.2830..	6.470	2	+0.01
.0190..	9.860	3	+ .11	.2550..	6.480	3	− .04
.0510..	9.158	4	.00	.2378..	6.563	3	.00
.0728..	8.248	5	− .05	.2270..	6.562	5	− .04
.0817..	8.033	3	.00	.2087..	6.730	3	+ .10
.0927..	7.809	7	.00	.1900..	7.000	2	+ .30
.1020..	7.590	4	− .08	.1777..	6.980	3	+ .11
.1128..	7.460	5	− .06	.1677..	6.928	4	− .02
.1215..	7.530	4	+ .13	.1540..	7.060	5	+ .04
.1312..	7.290	4	.00	.1385..	7.268	6	+ .08
.1407..	7.200	3	+ .05	.1277..	7.300	9	− .01
.1535..	7.005	2	− .05	.1170..	7.467	6	.00
.1643..	6.847	3	− .10	.1054..	7.526	5	− .09
.1795..	6.740	2	− .10	.0960..	7.712	6	− .03
.2067..	6.617	3	− .06	.0883..	7.839	10	− .05
.2320..	6.500	3	− .08	.0781..	8.108	8	− .01
.2390..	6.500	2	− .05	.0690..	8.380	11	− .04
.2518..	6.512	4	.00	.0566..	8.748	5	− .12
.2698..	6.500	4	+ .04	.0502..	9.160	6	− .01
.2805..	6.470	4	.00	.0348..	9.610	5	− .14
.3137..	6.447	3	+ .03	.0230..	9.792	4	+ .04
+ .3212..	6.485	4	− .06	−0.0017..	9.756	7	+0.01
−0.3087..	6.480	3	+0.06				

give these normals, with a correction of $+0^d.007$ already applied to the photographic phases, and $+0^d.010$ to the visual. This correction is explained below. Intensity-curves were drawn, and the

[1] *Mitteilungen der Hamburger Sternwarte*, Nr. 11 (1907).

[2] *Astronomische Nachrichter*. 200, 54, 1914.

best possible adjustment was made between the visual and photographic curves.

The visual duration of total eclipse appeared to be a little longer than the photographic, but as this seems to be physically incredible, a compromise was made, such geometric elements of the eclipse being used that the two light-curves were represented as well as could be done with elements identical in the two cases (except for the assumed brightness of the components). This solution assumes the stars to be uniformly luminous disks, and the eclipse central. The solution was made in the usual manner, Table IV being used

TABLE V

NORMAL MAGNITUDES OF U SAGITTAE—WENDELL

Phase	Mag.	No. of Obs.	O−C	Phase	Mag.	No. of Obs.	O−C
+0^{d}0059..	9.192	5	+0^{M}01	+0^{d}2353..	6.573	3	+0^{M}03
.0208..	9.173	6	− .01	.2548..	6.500	4	.00
.0330..	9.180	3	.00	− .2975..	6.480	2	+ .05
.0400..	9.180	2	+ .04	.2463..	6.610	3	+ .09
.0488..	8.970	4	+ .03	.2147..	6.697	3	+ .09
.0550..	8.730'	3	.00	.1863..	6.803	3	+ .04
.0617..	8.433	3	− .05	.1670..	6.920	4	.00
.0710..	8.140	2	− .02	.1460..	7.057	3	− .03
.0800..	7.953	3	.00	.1270..	7.270	4	− .01
.0902..	7.733	3	− .02	.1075..	7.480	2	− .01
.1095..	7.487	4	+ .01	.0800..	7.958	4	+ .01
.1333..	7.168	3	− .05	.0605..	8.560	5	+ .05
.1533..	7.040	3	+ .01	.0480..	8.958	4	+ .03
.1798..	6.825	4	.00	.0410..	9.140	4	+ .01
.1987..	6.677	3	− .03	.0255..	9.190	4	+ .01
+0.2182..	6.620	4	+0.02	−0.0108..	9.212	4	+0.03

(*Astrophysical Journal*, **36**, 252, 1912). From this solution we find the elements given in Table XVIII (p. 340).

The difference between the visual and photographic relative brightness of the two components was adjusted as for W Delphini; in this case the visual $(1-l)$ was multiplied by the constant factor, 1.0356. The visual range of light is from maximum 6.43 to minimum 9.18, giving a depth of 2^{M}75 or a loss of light of 0.9206. The photographic range is from maximum 6.42 to minimum 9.75, giving a depth of 3^{M}33 or a loss of light of 0.9534.

The light-curve thus derived was applied both to the visual and to the photographic normal observations. It was found necessary

to add $0^d.010$ to the visual phases, or, in other words, the epoch of minimum assumed was $0^d.010$ too late. In the same way the assumed epoch of minimum for the photographic observations was found to be $0^d.007$ too late. Least-square solutions were made to find more exactly the correction as described in the case of W Delphini. For the visual curve $x = +0^M.0111 \pm 0^M.0047$ and $y = -0^d.0006 \pm 0^d.00026$; that is, the minimum comes $0^d.0006$ earlier than the epoch, as already corrected. The solution of the photographic observations gives $x = +0^M.002 \pm 0^M.008$ and $y = +0^d.00044 \pm 0^d.00040$, or the photographic minimum is $0^d.00044$ later than the adopted time. Combining the two corrections each, for visual and photographic, we find that the visual minimum comes $0^d.0106$ early and the photographic $0^d.0066$ early, hence the photographic minimum comes $0^d.0040 \pm 0^d.00047$, or 5.8 ± 0.7 minutes later than the visual minimum. The probable error of one visual observation, as deduced from this solution, was $\pm 0^M.024$, and that of a photographic normal of unit weight depending on the average of 3.4 observations was $\pm 0^M.044$, corresponding to $\pm 0^M.091$ for a single plate. There seems to be no doubt that the difference, given above, between the visual and photographic minima, whatever its origin, is real.

The 349 observations falling outside the limits of the principal eclipse were collected into 14 normals, as shown in Table VI. These

TABLE VI

PHOTOGRAPHIC MAGNITUDES OF U SAGITTAE, OUTSIDE MINIMUM

Obs.	Phase	Mag.	Comp.	O−C	Obs.	Phase	Mag.	Comp.	O−C
26.....	$0^d.440$	6.416	6.413	$+0^M.003$	22....	$1^d.790$	6.439	6.451	$-0^M.013$
27.....	0.647	6.405	6.411	− .006	23....	1.956	6.426	6.432	− .006
32.....	0.843	6.403	6.410	− .007	26....	2.114	6.448	6.427	+ .021
22.....	1.074	6.428	6.411	+ .017	17....	2.336	6.402	6.429	− .027
27.....	1.265	6.403	6.413	− .010	33....	2.548	6.430	6.430	.000
20.....	1.460	6.418	6.416	+ .002	26....	2.740	6.426	6.429	− .002
18.....	1.635	6.441	6.442	−0.001	30....	2.953	6.435	6.423	+0.008

indicate that the light between eclipses is very nearly constant. The effects of ellipsoidal form of the stars and of the illumination of the side of the secondary which faces the primary are quite

insensible—which is not surprising, as the components are relatively widely separated.

The observations during the secondary minimum, which, according to Miss Fowler's spectrographic elements[1] should come at phase $1^d.744$, lie perceptibly below the mean of the others and are in entire accord with the theoretical depth of $0^M.031$ for uniform star-disks.

There appears, however, to be definite evidence that the star is brighter in the interval between the primary and secondary minima than in the other half of the period, as was found by Dugan[2] from his visual observations of RV Ophiuchi. The explanation suggested by Dugan—that the principal component is a little brighter on the preceding than on the following face—appears to be necessary here also, since the periastron, as determined from the spectrographic observations, falls in the half of the period when the star is fainter than in the other.

The expression for the magnitude which has finally been adopted is $6^M.420 - 0^M.010 \sin\theta$ plus the effects of eclipse, θ being the longitude in the orbit, measured from the principal eclipse. The computed magnitudes and residuals are given in Table VI. The probable error of one normal place comes out $\pm 0^M.0086$, which, since the average number of observations combined into a normal is 25, corresponds to a probable error of $\pm 0^M.043$ for a single photograph. This remarkably small value is confirmed by a study of the residuals of the individual observations, as will be shown later.

3. S CANCRI ($8^h38^m.2$, $+19°23'.6$)

S Cancri was handled in much the same way as the stars just discussed. Phases for the 365 observations, of which 20 were afterward rejected for large and unexplained discordances, were computed by the light-elements given by Shapley in *Contributions from Princeton University Observatory*, No. 3. These are 2403210.6053 +9.4845485 E, which are the same as given by Wendell,[3] except that the epoch of minimum here adopted is $0^d.02$ later. This period

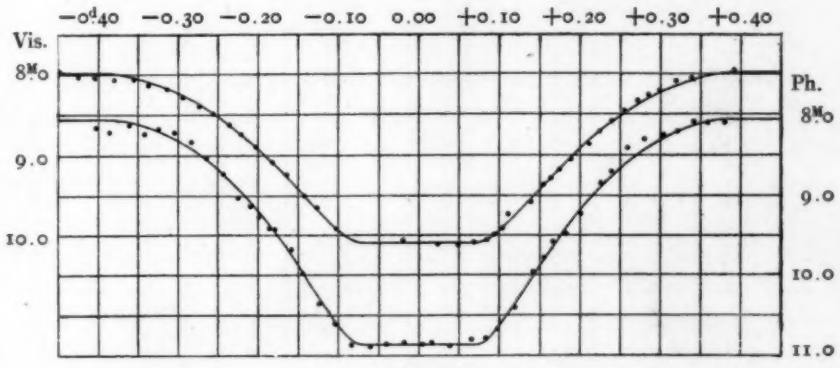
[1] *Publications of the Allegheny Observatory*, **3**, 11, 1912.

[2] *Astrophysical Journal*, **43**, 142, 1916. [3] *Harvard Annals*, **69**.

was adopted as definitive, and, as the exposures were all short, no correction to the magnitudes was necessary.

Maximum constant light from 35 observations is 8.06 and constant minimum light from 37 observations, 10.86, giving a range of $2^{M}80$, or a loss of light of 0.924 of the maximum intensity. In the visual curve as found by Shapley we have a range from 7.98 to 10.10, or $2^{M}12$, meaning a loss of light of 0.858.

The observations during eclipse were grouped into 43 normals and are given in Table VII. It was found that Shapley's curve, if



S Cancri

Visual, above; photographic, below

FIG. 3

merely altered to correspond to the photographic difference of magnitude of the two stars, did not satisfactorily represent the observations, but that a good agreement could be obtained by supposing that, while the disks appeared uniformly bright visually, they were darkened toward the limb for the photographically active wavelengths, so that the limb was half as bright as the center. The transformation of the curve was made with great ease and rapidity by means of Table VI, *Astrophysical Journal*, **36**, 403, 1912, in which values of $X(k, \alpha)$ are given, and the correction applied to the loss of light is, in this case, $\frac{1}{2}(1-l)X(k, \alpha)$. In the present case k is 0.370. The light-curve, thus altered, proved a perfectly satisfactory fit to the photographic observations, so no new solution was necessary. It was necessary, however, to make the time of

minimum for the photograph $0^d.010$ later. A least-square solution, similar to those made for W Delphini and U Sagittae, was made with the following result: $x = -0^M.012 \pm 0^M.0008$, $y = -0^d.0015 \pm 0^d.0008$. The two corrections being combined, the photographic minimum was found to be $0^d.0085 \pm 0^d.0008$, i.e., 12.2 minutes later than the

TABLE VII

NORMAL MAGNITUDES OF S CANCRI—HARVARD PHOTOGRAPHS

Phase	Mag.	No. of Obs.	O−C	Phase	Mag.	No. of Obs.	O−C
+0.0036..	10.87	5	+0.01	−0.4031..	8.16	5	+0.10
.0163..	10.84	3	− .02	.3862..	8.21	6	+ .15
.0382..	10.89	6	+ .03	.3620..	8.12	4	+ .03
.0654..	10.81	5	− .05	.3418..	8.24	6	+ .09
.0837..	10.80	4	− .03	.3245..	8.17	8	− .02
.0979..	10.68	8	+ .02	.3049..	8.22	9	− .05
.1194..	10.41	7	+ .06	.2844..	8.33	8	− .10
.1409..	9.97	7	− .05	.2666..	8.53	7	− .03
.1550..	9.79	3	− .01	.2431..	8.73	5	.00
.1658..	9.58	8	− .02	.2250..	9.03	10	+ .12
.1825..	9.49	8	+ .08	.2093..	9.13	8	+ .04
.2021..	9.23	9	+ .05	.1996..	9.26	7	+ .08
.2256..	8.86	5	− .04	.1867..	9.41	8	+ .05
.2397..	8.71	6	− .08	.1781..	9.42	11	− .03
.2597..	8.42	11	− .18	.1588..	9.68	9	− .06
.2808..	8.31	12	− .12	.1450..	9.98	6	+ .03
.3036..	8.25	10	− .05	.1242..	10.37	3	+ .09
.3216..	8.21	13	.00	.1014..	10.62	7	+ .01
.3415..	8.09	13	− .05	.0841..	10.87	5	+ .06
.3593..	8.11	11	.00	.0612..	10.90	5	+ .04
+0.3803..	8.11	7	+0.05	.0402..	10.86	6	.00
				−0.0179..	10.84	7	−0.02
0.419...	8.06	6	0.00	4.52....	8.05	5	−0.01
0.447...	8.12	6	+0.06	4.58....	8.06	5	0.00
3.87....	8.15	2	+0.09	9.061...	8.04	5	−0.02
4.44....	8.03	6	−0.03				

assumed minimum. A similar solution was also made for the visual curve as given by Shapley, with the result: $x = -0^M.002 \pm 0^M.003$, and $y = +0^d.00004 \pm 0^d.00041$, meaning that the visual minimum comes later than the assumed minimum by 0.1 ± 0.6 minutes. The photographic minimum therefore comes 12.1 ± 1.3 minutes later than the visual. The probable error of one visual observation of unit weight is $\pm 0^M.021$; that of a photographic normal, depending, on the average, on 8.1 observations, is $\pm 0^M.045$, and the corresponding probable error for a single photograph is $\pm 0^M.128$.

4. W URSAE MAJORIS ($9^h36^m.7$, $+56°25'$)

Miss Leavitt remarks: "On account of the brightness of this variable and the distance of the comparison stars used, the only plates on which it can usually be well observed are those taken with the 0.5-inch Ross-Zeiss and Voigtländer lenses." As exposures with the 0.5-inch telescopes are usually an hour in length, as in the case of U Sagittae, the recorded magnitudes do not represent the true brightness of the variable at the middle of the exposure. The mean length of exposure of all the observations was found to be 65.27 minutes. The visual light-curve given in *Astrophysical Journal*, **36**, 141, 1912, was smoothed out to allow for this mean exposure by interpolating the magnitudes to 5-minute intervals and taking means of these over intervals of 65 minutes, including as middle the specified time. It was then found that the photographic observations indicated a slightly greater range than this modified visual curve, the difference being $0^M.064$. The changes in the correction for the length of exposure on account of this fact amounted at most to only $0^M.004$, but have been applied. The corrections given in Table VIII were thus determined and applied eventually to the light-curve.

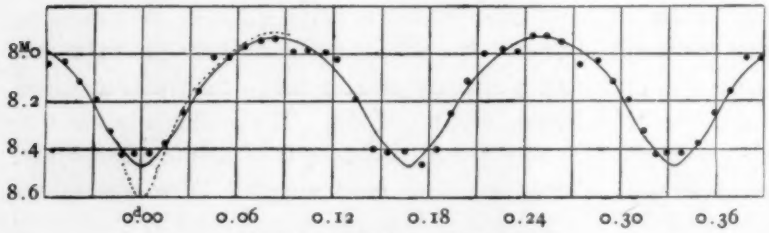
TABLE VIII

Phase	Correction	Phase	Correction
$0^d.0834$.....	$-0^M.02$	$0^d.0313$.....	$-0^M.04$
.0735.....	− .02	.0208.....	− .03
.0625.....	− .02	.0104.....	+ .07
.0520.....	− .01	0.0000.....	+ .13
0.0417.....	− .01		

Phases for the 367 observations were first computed with Baldwin's elements,[1] which are 2,416,129.1926+0.33364044 E. All the observations were divided into four groups, and times of minima were determined for each separately. As there seemed to be a systematic divergence from the assumed period, a careful investigation of all available observations was made, the results of which

[1] *Monthly Notices*, **69**, 86, 1908—doubling the period there given, as there are two minima in each orbital revolution.

appear in Table IX. These corrections to Baldwin's time of minimum were found, when plotted (with the exception of all of Müller and Kempf's observations), to lie nearly within their probable errors upon a straight line whose equation gives the correction of $+0^d.0092-0^d.00000142$ E to Baldwin's elements as adopted in computing preliminary phases. This correction gives as final elements 2,416,129.2018+0.33363902 E. Müller and Kempf's observations were found to lie on a line parallel to the line going through all the other observations, but at a distance of about $-0^d.006$. The epochs of minimum assigned to Müller and Kempf have all been corrected by $+2^m$ to allow for the difference between the form of



W Ursae Majoris—Photographic

FIG. 4

the light-curves assumed by these observers and in this paper (see *Astrophysical Journal*, **36**, 140, 1912). No explanation has been found for this remarkable systematic difference between their results and those of other observers, but it is evident that their observations, taken alone, are satisfied by the finally assumed period, though the actual epoch would have to be taken about 8 minutes earlier.

Phases were then corrected to this period, and normals were formed for each of the four groups separately. The observed normals were plotted, corrected for the influence of the length of exposure, and the resulting true photographic light-curve compared with the visual curve.

It was found that the portion of the variation due to eclipse was sensibly identical in the two cases, but that the variation outside eclipse, arising from the ellipticity of the stars, was greater photo-

graphically than visually. This can be accounted for by assuming that the star-disks appear to be darkened toward the limb to a greater degree in the former case than in the latter; for, if an ellipsoidal star whose eccentricity of the meridian section is ϵ

TABLE IX

MINIMA OF W URSAE MAJORIS

Authority	Method	Mean Epoch	Correction to Baldwin	O−C
Harvard	Photographic	560	+0.0079±0.0015	−0^d.0005
Parkhurst and Jordan*	Photographic	3,200	+ .004	− .001
Harvard	Photographic	3,690	+ .0033±0.0012	− .0007
Tikhoff†	Photographic (orange)	4,500	+ .002	− .001
Tikhoff	Photographic (ultra-violet)	4,500	+ .003	.000
Baldwin‡	Photometric	5,475	+ .0020± .0004	+ .0004
Horn§	Photographic	7,900	− .004	− .003
Harvard	Photographic	8,000	− .0012± .0008	+ .0010
Lazzarino‖	Photometric	9,286	− .003	.000
Harvard	Photographic	12,300	− .0086± .0011	− .0003
Müller and Kempf¶	Photometric	0	+ .0034± .0007	− .0058
Müller and Kempf	Photometric	111	+ .004	− .005
Müller and Kempf	Photometric	300	+ .002	− .007
Müller and Kempf**	Photometric	1,184	+ .001	− .006
Müller and Kempf	Photometric	2,200	− .000	− .006
Müller and Kempf	Photomertic	3,864	−0.003	−0.007

* *Astrophysical Journal*, **23**, 82, 1906.
† *Pulkova Mittheilungen*, **2**, 21, 1908.
‡ *Monthly Notices*, **69**, 78–90, 1908.
§ *Rivista di Astronomia*, July, 1910.
‖ *Astronomische Nachrichten*, **190**, 95, 1911.
¶ *Astrophysical Journal*, **17**, 231, 1903.
** *Monthly Notices*, **69**, 86, 1908.

revolves so that its equatorial plane is inclined to an angle ι, the variations of light due to its differing presentation are given by the equation

$$L^2 = L_0^2(1 - \epsilon^2 \sin^2 \iota \cos^2 \theta)$$

if it appears as a uniform disk, but by

$$L - L_0[1 - (\tfrac{4}{5}\epsilon^2 + \tfrac{16}{175}\epsilon^4) \sin^2 \iota \cos^2 \theta]$$

if the disk appears darkened toward the limb, and the range is greater in the second case (see *Astrophysical Journal*, **36**, 400, 1912).

In the present case it was found that the photographic light-curve outside eclipse could be excellently represented by the geometric elements derived from the visual observations, if the computed

light was taken halfway between the values obtained on the assumption of uniform disks and complete darkening—that is, if photographically the disks appear 50 per cent as bright at the limb as at the center. The changes in the variation due to eclipse resulting from this hypothesis were computed by Table VI, *Astrophysical Journal*, **36**, 142, 1912, as described in the case of S Cancri—the correction nowhere exceeding $0^{M}.011$.[1]

Solutions were made from each of the four groups of photographic observations, for magnitudes at maximum, and the correction to Baldwin's elements for the time of minimum, using the mean light-curve. The resulting times of minima are given in Table IX. The corrections to the assumed magnitude at maximum (7.912) came out -0.030 ± 0.010, $+0.012 \pm 0.004$, $+0.005 \pm 0.005$, and $+0.020 \pm 0.007$, respectively, for the four series, while the probable errors of a single observation came out $\pm 0^{M}.082$, $\pm 0^{M}.090$, $\pm 0^{M}.053$, and $\pm 0^{M}.068$. The small residuals found for the magnitude at maximum indicate that the star is not subject to sensible secular variations of brightness, and that the gradation and color-sensitiveness of the plates have been remarkably uniform from year to year.

In forming the final normals two observations with residuals exceeding $0^{M}.40$ were rejected, and the remaining 364 combined into 34 groups. The resulting normal magnitudes and phases are given in Table X, together with the number of observations combined into each normal, and the residuals from the final curve, which is derived from the theoretical curve for eclipses of stars partially darkened toward the limb, described above, by applying the corrections for the length of exposure given in Table VIII, with reversed sign, and hence represents the variation as it should be observed for exposure of 65 minutes' duration.

If these residuals are given weights proportional to the number of observations, the probable error of a single observation is found

[1] What the difference in the visual and photographic curves indicates is really that the *degree* of darkening toward the limb is greater in the latter case by 50 per cent of its maximum possible amount. Light-curves sensibly identical with those here given could be obtained, with slightly different geometrical elements, by the assumption that the loss of light at the limb was x per cent visually and $(x+50)$ per cent photographically, where x may have any value from 0 to 50.

to be $\pm 0^{M}.066$, from the first powers of the residuals, or $\pm 0^{M}.070$ from their squares—a very satisfactory showing.

TABLE X

W URSAE MAJORIS—FINAL NORMALS

Phase	Obs.	Mag.	O−C
$0^{d}.0049$	15	8.418	$-0^{M}.037$
.0148	17	8.376	− .002
.0253	11	8.246	− .032
.0352	10	8.157	.000
.0448	15	8.017	− .057
.0543	15	8.015	.000
.0638	11	7.968	− .001
.0744	13	7.942	+ .003
.0832	11	7.936	+ .004
.0948	14	7.993	+ .048
.1039	17	7.985	+ .011
.1150	9	7.997	− .033
.1222	6	8.022	− .056
.1338	5	8.190	+ .012
.1446	7	8.400	+ .088
.1532	5	8.414	+ .031
.1647	6	8.408	− .057
.1753	7	8.464	+ .033
.1844	5	8.400	+ .050
.1935	2	8.250	.000
.2038	6	8.117	− .024
.2137	3	8.000	− .058
.2254	5	7.980	− .014
.2351	17	7.993	+ .041
.2443	20	7.925	− .013
.2527	15	7.923	− .009
.2624	7	7.950	+ .005
.2743	7	8.043	+ .053
.2851	16	8.031	− .019
.2945	21	8.117	.000
.3048	13	8.192	− .040
.3141	11	8.323	− .014
.3214	10	8.420	+ .021
0.3289	12	8.417	−0.041

5. RW TAURI ($3^{h}57^{m}45^{s}$, $+27°51'$[1900])

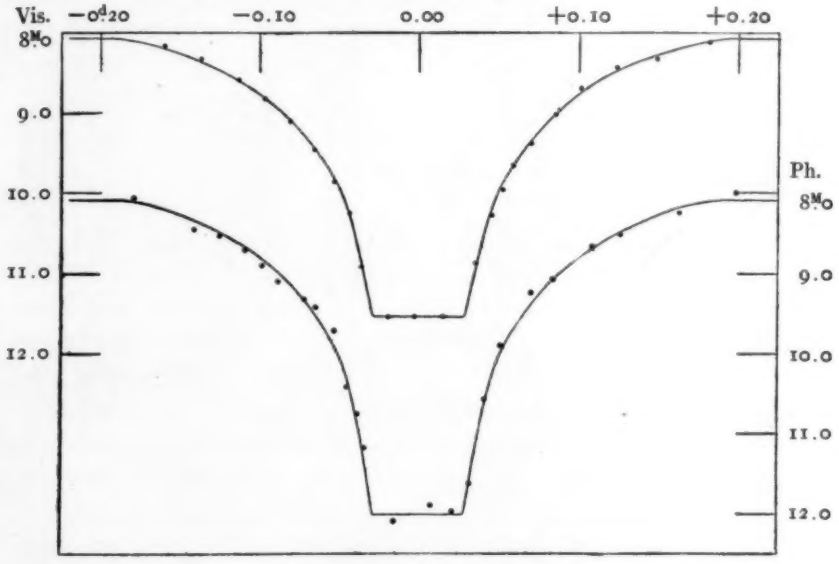
Material for the light-curve was obtained from the visual observations of Wendell,[1] and of Shapley,[2] and from the Harvard photographic observations, as previously described. Wendell's observations gave a well-defined curve; and, as Shapley did not have these at his disposal when determining his elements, a new light-curve

[1] *Harvard Annals*, **69**, 147, 1914.

[2] *Contributions from the Princeton Observatory*, No. 3, p. 136, 1915.

and corresponding set of elements were computed by combining Shapley's visual observations published in the volume just mentioned—taking the magnitude of his comparison star as 8.88, Wendell's observations—using Shapley's period, slightly lengthened to $2^d.768{,}903$ and his epoch 2,417,198.4094, which is $0^d.129$ earlier than that of Wendell.

Two of Wendell's observations were rejected, as it was evident some error had been made. From 38 observations by Wendell and



RW Tauri

Visual, above; photographic, below

FIG. 5

7 by Shapley the maximum was determined as 8.07; and from 14 by Wendell and 6 by Shapley the constant light at minimum was found to be 11.54, making the range $3^M.47$. In solving for the elements on the assumption of uniformly luminous star-disks it was found that the assumption of a central total eclipse led to the elements given in Table XVIII, and an excellent representation of the observations, as shown in Table XI.

The period derived from the visual observations represents the photographic data from 1899 to 1915 in a very satisfactory manner;

but a few observations in earlier years show large discrepancies. The observed minima came earlier than those computed by $0^d.07$ in 1891 (J.D. 1782), and by about $0^d.10$ in 1888 (J.D. 0954) and $0^d.14$ in 1886 (J.D. 9913), the last two photographs being taken during totality, so that the exact correction is uncertain.

TABLE XI

RW TAURI—VISUAL NORMALS

Phase	Mag.	Obs.	O−C	Phase	Mag.	Obs.	O−C
−0.1601..	8.162	8	−0.038	+0.0152..	11.538	6	−0.002
.1370..	8.334	7	− .026	.0354..	10.864	5	− .066
.1131..	8.589	7	− .001	.0452..	10.290	5	+ .110
.0971..	8.811	7	+ .011	.0523..	9.971	6	+ .106
.0811..	9.100	7	+ .010	.0584..	9.657	7	.000
.0666..	9.456	7	− .014	.0700..	9.380	8	+ .040
.0530..	9.862	6	+ .022	.0850..	9.013	7	+ .013
.0434..	10.267	6	− .048	.1012..	8.682	5	− .058
.0362..	10.912	5	+ .012	.1233..	8.433	7	− .042
.0194..	11.541	8	+ .001	.1488..	8.330	6	+ .055
0.0023..	11.540	6	0.000	0.1818..	8.110	5	+0.010

These observations can be represented by the assumption that the period previous to 1895 was about 10 seconds shorter than subsequent to that year; but the data are too scanty to determine the true nature of the change in period, although its reality seems to be unquestionable. These early observations were omitted from the further discussion.

The remaining observations were plotted, and it was found that many of those made while the star was changing rapidly gave values of its light which were obviously half a magnitude or more too bright. This was soon recognized as the effect of the length of exposure. The star at certain times varied by two or even three magnitudes during the longer exposures, and the corrections on this account are of unusual magnitude.

It being assumed, as usual, that the photographic action was proportional to the intensity of the light, it was found that, outside the constant phase of minimum, the curve giving the intensity as a function of the time was nearly linear, so that the intensity at mid-exposure was substantially equal to the mean value during the whole time of exposure, and no correction was necessary. But

when part of the exposure fell during the constant phase, the results are very different. From a provisional curve based on the plates with short exposure, the intensity at $9^{M}.50$ being taken as 1000, it was found that the actual intensity was 100 from phase $0^{d}.000$ to $0^{d}.028$ and beyond this point was represented closely enough, though not exactly, by the formula

$$I = 100 + 25.6(t - 31) \tag{1}$$

where t is measured in thousandths of a day from mid-eclipse.

Hence, for an exposure extending from t_1 to t_2, where t_1 is less and t_2 greater than 31, the mean intensity is easily found to be

$$M = 100 + \frac{12.8(t_2 - 31)}{t_2 - t_1} \tag{2}$$

For observations on the descending branch $t_1 + 31$ takes the place of $t_2 - 31$. If I_0 is the value of I at the middle of the exposure, as given by formula (1), I_0 being 100 if $\frac{1}{2}(t_2 + t_1)$ is numerically less than 31, then the difference in stellar magnitude corresponding to the ratio M/I_0 is evidently the correction desired. This should always be *added* to the observed magnitude to obtain the magnitude which would have been found from a short exposure.

The computation of these corrections is summarized in Table XII, which needs no further explanation except to say that the first column contains the serial number of the observation concerned in the detailed list (to be published in the *Harvard Annals*) to facilitate its identification there.

The observed magnitudes, so corrected, were combined into normals; 7 observations which made the star more than half a magnitude brighter or fainter than was indicated by the mean curve were rejected.

It was found that a very satisfactory representation of the photographic observations could be obtained by using the geometrical elements derived from the visual solution, but by taking the photographic magnitudes as 8.09 at maximum (derived from 31 observations outside the eclipse) and 12.00 at minimum (from 12 observations) and displacing the curve in time so that the middle of eclipse came $0^{d}.0025$ earlier photographically than visually.

These normals and the residuals (O−C) from the curve are given in Table XIII. The representation is quite as satisfactory

TABLE XII

No.	Phase $\frac{1}{2}(t_1+t_2)$	t_1 or t_2	M	I_o	Correction	Obs. Mag.	Corr. Mag.
8.....	−0.029	−52	223	100	0.87	10.45	11.32
16.....	+ .046	+95	642	484	0.31	10.58	10.89
19.....	− .018	−40	124	100	0.23	11.65	11.88
30.....	+ .032	+69	350	126	1.11	10.55	11.66
37.....	− .034	−49	238	177	0.32	10.95	11.27
48.....	+ .010	+33	101	100	0.01	11.85	11.86
50.....	+ .014	+47	150	100	0.44	11.55	11.99
51.....	− .020	−43	140	100	0.37	10.95	11.32
53.....	+ .029	+57	255	100	1.04	11.57	12.61
62.....	+ .028	+52	218	100	0.85	10.90	11.75
64.....	− .038	−59	339	280	0.20	10.40	10.60
70.....	+ .043	+71	466	408	0.14	10.11	10.25
71.....	− .050	−72	590	587	0.01	9.95	9.96
121.....	+ .024	+45	160	100	0.51	11.65	12.16
125.....	+ .041	+62	393	356	0.11	10.50	10.61
128.....	− .051	−73	613	612	0.00	9.75	9.75
136.....	+ .042	+61	404	382	0.06	10.45	10.51
106.....	−0.035	−42	209	202	0.04	11.05	11.09

TABLE XIII

RW TAURI—PHOTOGRAPHIC NORMALS

Phase	Obs.	Mag.	O−C	Phase	Obs.	Mag.	O−C
−0.179...	4	8.07	−0.05	−0.015...	3	12.09	+0.09
.141...	4	8.46	+ .10	+ .008...	3	11.89	− .11
.125...	4	8.54	+ .04	.022...	3	11.97	− .03
.109...	5	8.71	+ .03	.032...	2	11.61	+ .02
.096...	6	8.90	+ .04	.041...	4	10.56	− .03
.088...	6	9.10	+ .10	.051...	2	9.90	− .10
.072...	5	9.32	− .01	.070...	3	9.24	− .15
.065...	3	9.42	− .11	.089...	4	8.97	− .01
.053...	6	9.71	− .21	.108...	5	8.67	− .02
.045...	2	10.42	+ .12	.126...	3	8.52	+ .03
.038...	2	10.75	− .03	.163...	3	8.25	+ .05
−0.034...	2	11.18	−0.12	+0.199...	1	8.00	−0.09
0.56....	6	8.08	−0.01	1.80....	5	8.01	−0.08
1.05....	5	8.09	.00	2.18....	5	8.15	+ .06
1.36....	4	8.04	−0.05	2.46....	6	8.13	+0.04

as could possibly be expected under the circumstances, the probable error of a single observation, as derived from the weighted sum of the square of the residuals, being 0.111. The 31 observations

outside eclipse yield a smaller probable error, $\pm 0^{M}076$, so that the "brute mean" probable error for all the 116 observations of this star is $\pm 0^{M}099$.

In addition to these plates, 13 others showed no image of the star, proving it to be fainter than specified limiting magnitudes. In all cases the computed magnitude was well below the limit.

The photographic range of this star, $3^{M}91$, is the greatest range so far observed in any eclipsing variable.

6. SW CYGNI ($20^{h}03^{m}50^{s}$, $+46°0'.5$ [1900])

The material for the study of this star consists of 192 visual observations by Wendell,[1] and of 286 estimates from Harvard photographs.

The elements derived from the visual observations were 2,411,343.630+4.572908 E, the period being the same as that adopted by Graff and Shapley,[2] but the epoch $0^{d}029$ later than Shapley's.

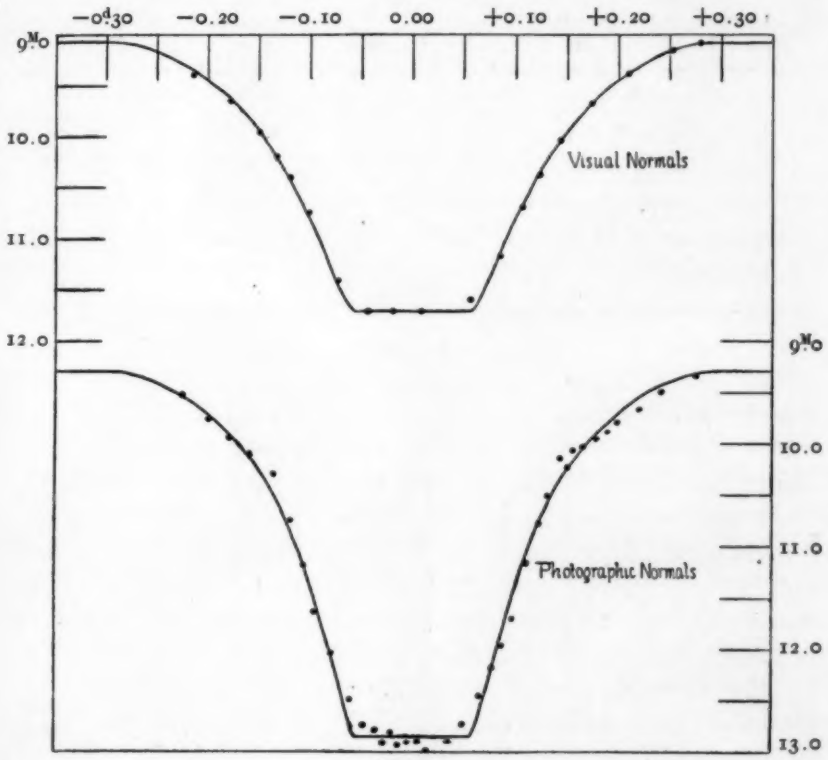
The visual observations (with the exception of five which showed large discordances) were plotted with Shapley's theoretical "uniform" curve, which proved to be an excellent fit, as is shown by the residuals given in Table XIV. The maximum is 9.06 and minimum 11.72. A least-square solution gave a correction to the assumed epoch of minimum of $-0^{d}0040 \pm 0^{d}0004$, which has been applied to the phases given in Table XIV.

The photographic material consists of 220 observations down to magnitude 12.5, from estimates made in 1903, reduced with magnitudes of the comparison stars furnished by Miss Leavitt in 1916, and of 66 estimates of the fainter magnitudes made by Miss Leavitt in 1916. Only two observations of the former group had to be rejected for large discordances, exceeding one magnitude. The remainder were combined into 37 normals, as indicated in Table XV. The residuals there given are derived from a curve computed from the same geometrical elements as Shapley's visual curve, but with the magnitude at maximum 9.29 and at minimum 12.85. The latter value was well determined by 79 observations during the

[1] *Harvard Annals*, **69**, 83, 161, 1892–1912.

[2] *Astrophysical Journal*, **36**, 269, 1912.

total eclipse. As there were only six observations outside eclipse, the magnitude at this phase was determined mainly from the observations on the upper parts of the curve, where the eclipse is partial and small.



SW Cygni

FIG. 6

The representation of the observations by the curve is less satisfactory than for the other stars, the residuals being large and positive about 0d10 on each side of minimum, and large and negative about 0d14 on either side of the middle. Though a freehand curve would represent the observations much better than the curve here given, such a curve would have to cross the observed curve three times during the ascent or descent; and, since only three parameters are available in fixing the shape of an eclipse curve, this

means that the freehand curve would be of a shape impossible to represent on the eclipse hypothesis.

TABLE XIV

SW CYGNI—VISUAL NORMALS

Phase	Mag.	Obs.	O−C	Phase	Mag.	Obs.	O−C
−0d215...	9.38	10	+0M08	+0d009...	11.72	10	−0M01
.178...	9.65	11	+ .02	.056...	11.60	10	− .13
.150...	9.95	10	+ .05	.086...	11.17	10	+ .05
.133...	10.18	10	.00	.106...	10.69	11	+ .02
.120...	10.40	10	+ .01	.123...	10.37	11	+ .04
.102...	10.74	9	− .02	.143...	10.05	10	+ .01
.073...	11.41	10	− .03	.174...	9.67	10	− .01
.044...	11.72	10	− .01	.210...	9.38	10	.00
−0.019...	11.71	10	−0.02	.253...	9.15	10	.00
				+0.280...	9.09	5	0.00

TABLE XV

SW CYGNI—PHOTOGRAPHIC NORMALS

Phase	Mag.	Obs.	O−C	Phase	Mag.	Obs.	O−C
−0d224...	9.52	7	−0M01	+0d049...	12.73	9	−0M12
.198...	9.75	8	+ .03	.065...	12.41	8	− .18
.179...	9.94	8	+ .04	.077...	12.17	7	− .01
.158...	10.10	7	− .03	.087...	11.96	7	+ .10
.134...	10.29	7	− .21	.096...	11.70	8	+ .22
.119...	10.74	9	− .08	.109...	11.16	7	+ .04
.106...	11.17	7	.00	.122...	10.78	6	+ .04
.096...	11.63	8	+ .15	.131...	10.51	7	− .06
.078...	12.04	9	− .10	.143...	10.14	7	− .22
.062...	12.49	9	− .25	.150...	10.23	7	− .01
.048...	12.74	9	− .11	.157...	10.07	7	− .09
.036...	12.78	8	− .07	.166...	10.03	8	− .01
.028...	12.91	8	+ .06	.178...	9.95	8	+ .05
.021...	12.81	8	− .04	.190...	9.89	7	+ .10
.013...	12.93	8	+ .08	.199...	9.78	7	+ .07
− .005...	12.91	8	+ .06	.221...	9.66	7	+ .12
+0.003...	12.90	7	+ .05	.243...	9.48	7	+ .06
.014...	12.99	7	+ .14	+0.276...	9.32	5	.00
+0.034...	12.90	7	+0.05	3.732...	9.14	6	−0M15

The principal discordances, already mentioned, are such as would arise if the assumed magnitudes of the comparison stars were somewhat erroneous. Whether this be the true explanation or not, these discordances, since they are symmetrical on each side of the minimum, will have no sensible influence on the computed time of mid-eclipse.

A least-square solution gave, as correction to the assumed epoch of minimum (1343.630), the value $+0^{d}.0002 \pm 0^{d}.0008$. The probable error of one normal place, depending, on the average, upon 7.5 observations, comes out $\pm 0^{M}.072$, which makes one probable error of a single observation $\pm 0^{M}.196$. This is much greater than in any of the cases previously discussed—mainly on account of the systematic deviations already mentioned, but partly because the individual observations are in any case mutually discordant. The residuals from a freehand curve lead to a probable error of $\pm 0^{M}.130$ for a single observation.

From the visual and photographic solutions it appears that the photographic minimum is later than the visual by $0^{d}.0042 \pm 0^{d}.0009$ —which, in view of the number and quality of the observations concerned, appears to be unquestionably real.

DISCUSSION OF THE RESULTS

A. *Errors of the photographic observations.*—At this stage of a work which it is hoped may prove of considerable extent, it is well to consider with some care the degree of precision which the observations possess. The individual residuals for all the observations of each star have therefore been arranged in order of their numerical magnitude, and the numbers between given limits compared with the theoretical law of distribution corresponding to the probable error computed in the ordinary fashion. The principal results of this inquiry are presented in condensed form in Table XVI.

For each star there are first given the numbers of positive and negative residuals between the limits specified at the left (residuals of a limiting value, such as $0^{M}.05$, being equally divided between the preceding and following groups). Following these are two columns, the whole number of residuals, regardless of sign, between the assigned limits, and the number which might be expected on the Gaussian theory, with the probable errors given below. The observations of U Sagittae during and outside the principal eclipse are listed separately, as they are of different degrees of precision.

The lowest line of this part of the table gives the whole number of positive and negative residuals. The table so far deals only with those observations which were finally admitted into the discussion.

TABLE XVI

RESIDUAL ERRORS OF THE PHOTOGRAPHIC OBSERVATIONS

LIMITS	S CANCRI				W DELPHINI				SW CYGNI			
	Pos.	Neg.	All	Theory	Pos.	Neg.	All	Theory	Pos.	Neg.	All	Theory
$0^{M}.00$ to $0^{M}.05$	40	52	92	85	53	47	100	98	25	33	58	56
.05 to .10	38	50	88	79	48	49	97	88	31	23	54	53
.10 to .15	30	21	57	64	38	31	69	72	22	19	41	46
.15 to .20	22	24	46	49	30	20	50	54	19	21	40	39
.20 to .30	24	22	46	50	27	23	50	57	26	24	50	52
.30 to .40	9	8	17	14	9	5	14	17	13	16	29	25
.40 to .50	2	3	5	4	2	3	5	3	6	4	10	9
Over 0.50	.....	.....	.....	.....	.....	5	5	1	.....	2	2	4
Total	165	180	345	(345)	207	183	390	(390)	142	142	284	(284)
Rejected	3	17	20	.....	8	3	11	.....	0	2	2	.....
Limit	$0^{M}.40$				$0^{M}.55$				$0^{M}.60$			
Average	0.56				1.07				1.09			
Probable error:												
First powers	$\pm 0^{M}.105$				$\pm 0^{M}.107$				$\pm 0^{M}.136$			
Squares	± 0.107				± 0.111				± 0.136			
Normals	± 0.128				± 0.132				± 0.196			

Limits	RW Tauri				W Ursae Majoris				U Sagittae							
									During Eclipse				Outside Eclipse			
	Pos.	Neg.	All	Theory	Pos.	Neg.	All	Theory	Pos.	Neg.	All	Theory	Pos.	Neg.	All	Theory
0.00 to 0.05	16	18	34	32	65	87	152	139	35	39	74	70	112	96	208	209
.05 to .10	14	20	34	28	47	52	99	109	34	30	64	58	72	28	100	108
.10 to .15	7	14	21	22	33	38	71	67	18	16	34	38	6	20	26	28
.15 to .20	4	4	8	15	11	14	25	32	5	8	13	21	2	7	9	4
.20 to .30	9	3	12	15	7	7	14	16	4	7	11	13	1	5	6	0
.30 to .40	2	3	5	3	3	.....	3	1	3	1	4	2	.....	.....	.....	.....
.40 to .50	.....	2	2	1	.....	.....	.....	.....	1	1	1	0	.....	.....	.....	.....
Over 0.50	.....	.....	.....	.....	.....	.....	.....	.....	.....	.....	.....	.....	.....	.....	.....	.....
Total	52	64	116	(116)	166	198	364	(364)	100	102	202	(202)	193	156	349	(349)
Rejected	2	5	7	.....	0	2	2	.....	4	4	8	.....	1	0	1	.....
Limit	0.50				0.40				0.50				0.30			
Average	0.89				0.52				1.76				1.16			
Probable error:																
First powers	±0.094				±0.068				±0.075				±0.040			
Squares	±0.098				±0.068				±0.079				±0.045			
Normals	±0.099				±0.070				±0.091				±0.043			

The next three lines give particulars concerning the observations rejected on account of inadmissibly large discordances: the first giving the number of those with positive and negative residuals and the whole number rejected; the second, the limit of rejection[1] (observations with residuals numerically exceeding this being rejected); and the third, the average value, regardless of sign, of the rejected residuals. It is clear that these very large residuals cannot have arisen from accidental errors of the same character as those of the bulk of the observations. Some of them may be due to defects of the plates not obvious on inspection; but it is probable that others arise from mistakes of some sort. In extensive routine work like the present, where hundreds of observations of the same star are available, the best course to pursue with regard to such discordant observations is clearly to reject them summarily. To spend an exceptional amount of work upon these doubtful observations—on the chance that in some cases the mistake, whatever it is, may be rectified—would demand so great an expenditure of time and money, in proportion to the increase of accuracy which might conceivably be attained in the final results, that this policy would be utterly inconsistent with the principles of "scientific management."

The percentage of observations so rejected varies from 0.6 for W Ursae Majoris to 5.5 for S Cancri and to 5.7 for RW Tauri. For the second of these stars the limit of rejection was set relatively low in proportion to the probable error later determined. For the third, four out of the seven rejected plates are long exposures made while the star was very faint and varying rapidly, under which conditions changes in the transparency of the atmosphere during the exposure may introduce large errors.

In the foregoing list of rejections no account is taken of plates recorded as unsatisfactory by the measurer, or of those on which the star was invisible, being near minimum.

The last three lines in Table XVI give for each star the probable error of a single photographic observation, as determined (1) from the numerical average of the first powers of the individual residuals,

[1] For S Cancri the assigned limit is only approximate, as no rigorous rule of rejection was followed in this, the first case discussed.

(2) from the squares of these residuals, and (3) from the squares of the residuals for the normal points previously given. The values found in the first two ways are always very nearly equal. Those found from the normals are often perceptibly greater, but the difference is serious only in the case of SW Cygni, where the existence of systematic deviations from the computed curve has already been remarked. In this case the substitution of a smooth and symmetrical freehand curve reduced the computed probable error of a normal point from $\pm 0^{M}.072$ to $\pm 0^{M}.048$. If, as is probable, the square of the probable error of an observation, derived from the individual residuals, would be diminished by the same amount, it would become $\pm 0^{M}.126$, as against $\pm 0^{M}.130$ derived from the discordance between the normal points and this same curve.

Upon examination of Table XVI it is noticeable that the comparison of the observed with the theoretical distribution of the residuals shows usually an excess of small and large values, with a deficiency of those of intermediate size. This indicates, as is well known, that the observations, even though the seriously discordant ones have already been rejected, are not all of the same precision—the best observations furnishing the excess of small residuals, and the worst the superfluity of large ones.

The estimates made recently at Harvard are much more precise than those made in 1903, the average probable error (as derived from the individual residuals, weighting the various series in proportion to the number of observations) being $\pm 0^{M}.068$ for the former and $\pm 0^{M}.117$ for the latter. The recent observations, which are probably a good indication of what may be expected in future work, show that the Harvard photographs, though taken with no particular reference to the variables here studied, or, indeed, to photometric work at all, give results of a very considerable degree of precision.

When the great number of photographs available is considered, it is evident that the fund of information contained in the Harvard Photographic Library is of very great value and that the discussion of as much of this material as possible is urgently to be desired.

B. *Periods and epochs of eclipse.*—The periods and epochs of mid-eclipse which were finally adopted for the stars here discussed

are given in Table XVII. The upper part of this table gives the elements for prediction of the visual minima in the ordinary form. The irregularity in the period of W Delphini, here represented by a quadratic term, is very likely to turn out ultimately to be periodic, and predictions by the formula here given should therefore be made with caution.

TABLE XVII

VISUAL MINIMA

Star	J. D.			
W Delphini	2412002.5594+4.806211 E−0.000000062 E^2			
U Sagittae	2417130.4027+3.380621 E			
S Cancri	2413210.6053+9.4845485 E			
W Ursae Majoris	2416129.2022+0.33363902 E			
RW Tauri	2417198.4094+2.768903 E			
SW Cygni	2411343.6260+4.572908 E			
	Phot.–Vis. P.E.	Duration of Increase	Distance Light-Years	ΔV Meters/Second
W Delphini	+0^d0026	0^d264	2900	+0.7
U Sagittae	+ .0040±0^d0005	.262	960	+3.4
S Cancri	+ .0085±0.0009	.315	1370	+5.1
W Ursae Majoris	− .0004±0.0007	.034	165	−2.0
RW Tauri	− .0025	.175	2200	−0.9
SW Cygni	+0.0042±0.0009	0.230	2400	+1.4

The lower half of the table contains data relating to the difference in time between the moments of mid-eclipse, as determined from the visual and photographic observations.[1] The second column gives this difference, which is positive if the photographic minimum comes late; and the third gives the probable error of this quantity (taking into account the errors of both the visual and photographic observations) as derived in the cases where least-square relations were made. For the two cases in which no such solutions were made the probable errors are not given, but the determinations are of about the same accuracy as the others.

In five of the six cases these differences are unquestionably real, and light-curves with the same epoch of mid-eclipse cannot give a satisfactory representation of both the visual and photographic observations. Similar differences in the times of minimum, as

[1] Baldwin's visual observations in the case of W Ursae Majoris. See p. 320.

observed with different wave-lengths, have often been reported before; but the evidence of their reality is in the present case far more conclusive than previously, depending as it does upon hundreds of observations of each star, at practically the same epochs for the visual and photographic work (the former always falling within the interval covered by the latter), which have been reduced with the aid of light-curves whose form is not arbitrary, but is determined by a well-tested theory.

The origin of these differences however remains decidedly obscure. The most obvious suggestions are (*a*) that the velocity of the visual and photographic rays in interstellar space is slightly different, as suggested by Nordmann, and (*b*) that the distribution of brightness over the surface of the star-disks is uneven, and different for the different wave-lengths, as suggested by Lebedew.[1]

The fact that the observed differences are not always of the same sign is strongly against the first of these explanations. Moreover, the distances of the various stars can now be approximately estimated, in the manner described by Russell and Shapley.[2] These distances are given in the fifth column of Table XIX, the data for RW Tauri and U Sagittae being recalculated to correspond to the new elements of the present paper. The difference in velocity of the visual and photographic rays which would be required to explain the observed discrepancy for each star is given in *meters* per second in the last column of Table XVII. The various results are utterly discordant and do not furnish the slightest evidence in favor of the hypothesis. It is, however, decidedly interesting to note that the observations of even a single star prove the identity of the velocity of light of the different wave-lengths within a few meters per second, that is, on the average, to within a hundred-millionth part of the whole.

Probably no other direct observational test is of comparable delicacy; but since a difference of velocity of even one meter per second would demand that the medium occupying interstellar space should have a dispersive power of about one fifteen-hundredth that of ordinary air at the earth's surface, there is good indirect reason

[1] *Astrophysical Journal*, **29**, 101, 1908.

[2] *Ibid.*, **40**, 417–434, 1914.

to suppose that, if any such influence really exists, its effects must be far too small to measure, even in this way.

The second explanation appears more probable, especially as the amount of shift of the minimum is always but a small fraction of the whole time occupied by the increase or decrease of light during the eclipse (which is given in the fourth column). But in the case of S Cancri the shift is one thirty-seventh of the whole time of increase, which seems a rather large amount to be accounted for by a *difference* in the manner of distribution of yellow and violet light over the star-disk. Again, in the case of U Sagittae, when the visual light of the brighter star is half eclipsed (visual magnitude 7.10), the photographic magnitude is 7.08 during the decreasing, and 7.16 during the increasing, phase. At these times the visible portions of the bright component are practically the two opposite halves of the disk (since the eclipse is central), and it is evident that these two regions must differ in color-index by at least $0^{M}.08$ to explain the observations in this way—the preceding half of the disk being the redder. But, when these opposite faces of the star are turned toward us a quarter-period before and after eclipse, the difference of magnitude is only $0^{M}.02$, and the preceding face appears to be photographically brighter than the other.

Wendell has four visual observations in the interval between primary and secondary minimum, with a mean magnitude of 6.415, and nine in the other half of the period, with a mean of 6.429. It appears, therefore, very doubtful that differences in color-index between the opposite sides of the star, if present at all, can be great enough to account for the observed retardation of the photographic minimum.

A third explanation deserves mention. It has been shown by Pickering that, when several successive exposures are made on one plate, the plate usually appears to be more sensitive for the first exposure than for the others. Hence, when a star of varying brightness is photographed, the effective mean instant of exposure, this variation of sensitiveness being taken into account, will be earlier than the middle of the exposure. A simple calculation shows that, if the sensitiveness during the first half of the exposure were 10 per cent greater than the average and that in the second half 10 per cent

less, the effective mean time of exposure for a star varying in brightness at a uniform rate would be earlier than the middle of exposure by one-twentieth of the whole duration of exposure. If no allowance is made for this, the photographic variation will appear to come late, the shift being $0^d.002$ for exposures of an hour. This may have something to do with the tendency of the observed photographic minima to come late, but can hardly explain the case of S Cancri, in which the shift is 12 minutes, and the average length of exposure between 20 and 30 minutes.

For the present it appears necessary, therefore, to leave the origin of these small shifts as a problem for future solution.

C. *Astrophysical results.*—1. The present investigation furnishes the most comprehensive test which has yet been made of the theory of eclipsing variables. Two independent, extensive, and precise series of observations were available for each star, and these have in all cases been satisfactorily represented by exactly the same geometrical elements—size and shape of the components and inclination of the orbit—only the brightness of the components and, in some cases, the degree of darkening toward the limb being different for the visual and photographic observations. The representation of the latter by the elements derived from the visual observations is excellent for all the stars but SW Cygni, and fair in this case, in which the discrepancies can reasonably be attributed to errors in the early and less precise photographic estimates. Table XVIII gives the elements of the six systems. The geometrical elements are first given, those of U Sagittae and RW Tauri being derived from the computations described above and the others being taken from Shapley's list.[1] These are in all cases the "uniform elements" derived on the assumption that the visual brightness of the star-disks is everywhere the same. For S Cancri and W Ursae Majoris the photographic brightness appears to be only half as great at the edge of the disks as at the center. The stars are spherical (or at least spheroidal) except in the case of W Ursae Majoris, where they are ellipsoidal, the two shorter radii being 0.745 and 0.681 times the greatest radius given in the table. The radii, as tabulated, are expressed as fractions of the radius of the relative orbit.

[1] *Contributions from the Princeton University Observatory*, No. 3, pp. 82–85.

The visual and photographic ranges of variation are next given, the former being taken in most instances from Shapley's work. The latter is invariably the greater, showing that the stars are redder at minimum than at maximum—and usually considerably so.

TABLE XVIII

ELEMENTS OF ECLIPSING BINARIES

	W Delphini	U Sagittae	S Cancri	RW Tauri	SW Cygni	W Ursae Majoris
Radius of bright star	0.135	0.230	0.075	0.190	0.131	0.366
Radius of faint star	0.256	0.295	0.203	0.254	0.266	0.366
Inclination of orbit	83°27′	90°0′	83°10′	90°0′	83°42′	77°32′
Visual range	$2^{M}.70$	$2^{M}.75$	$2^{M}.12$	$3^{M}.47$	$2^{M}.66$	$0^{M}.60$
Photographic range	3.32	3.33	2.80	3.91	3.56	0.68
Bright component:						
Visual magnitude	9.49	6.51	8.14	8.11	9.16	(8.66)
Photographic magnitude	9.48	6.47	8.14	8.12	9.33	8.68
Color-index	−0.01	−0.04	0.00	+0.01	+0.17	(+0.02)
Spectrum	A	B8	A	B5	A	G
Density	0.18	0.073	0.25	0.20	0.21	2.45
Faint component:						
Visual magnitude	12.10	9.18	10.10	11.54	11.72	(8.66)
Photographic magnitude	12.75	9.75	10.86	12.00	12.85	8.68
Color-index	+0.65	+0.57	+0.76	+0.46	+1.13	(+0.02)
Density	0.008	0.011	0.005	0.024	0.009	2.45

Following this appear data concerning each component separately—the visual and photographic magnitudes, the color-index, the spectral class (which is known only for the brighter components), and the density—in computing which the relative masses of the components have been estimated on the basis of their visual brightness.[1]

These visual magnitudes have been determined by comparison with stars which appear in some photometric catalogues (usually only one star in each case), and the photographic magnitudes by comparison with the Polar Sequence. The zero-points to which the magnitudes of the different stars are referred (especially the visual magnitudes) may be subject to small errors, and too much weight should not be laid upon the absolute amounts of the resulting color-indices. For W Ursae Majoris, in particular, the visual magnitude is given on the Potsdam scale and requires a large correction—

[1] *Contributions from the Princeton University Observatory*, N . 3, pp. 121–123.

probably about $-0^{M}.3$—to reduce it to the Harvard scale, so that the color-index given in the table is not comparable with the others.

The *scale* of magnitudes, however, should in all cases be well determined, and the relative color-index of the two components of each system should be trustworthy.

2. All these stars except W Ursae Majoris have been selected from a well-defined class—those with total eclipses, large range, and components markedly unequal in brightness, the smaller star being much the brighter and denser of the two.

The brighter components of these five systems are very similar, their average spectrum being B 9, their average color-index $+0^{M}.03$ —in very good agreement with the spectral class—and their average density 0.18 that of the sun. They appear to be entirely typical of the first-type stars in general. The fainter components are also very similar *inter se*, but are of much lower density, averaging only 0.011 that of the sun, and are all far redder than their primaries, their average color-index being $+0^{M}.72$. If the relation between color-index and spectrum is the same for these point-images of faint stars as for the extra-focal images of brighter ones upon which the ordinary system is based, this would correspond to the spectral class G 0; but there is evidence that in this case the color-equation is smaller than the standard value, so that the given difference of color-index would indicate an even greater difference of spectral type.

In any case it is clear that these large faint companions are *second-type stars*. Direct observations of their spectra during the constant phase at minimum should be possible in some cases at least and are much to be desired.[1]

Similar differences in color-index, always in the same sense, have previously been reported by Seares[2] and Shapley[3] in the case of RR Draconis, by Shapley for Y Piscium[2] and TW Andromedae,[4] and by Dugan for RZ Cassiopeiae.[5] The additional results here

[1] In the very similar system U Cephei, the spectrum of the companion is K (Miss Cannon, *American Astronomical Society*, December meeting, 1916).

[2] *Astrophysical Journal*, **36**, 368, 1912. [3] *Ibid.*, **37**, 160, 1913.

[4] *Publications of the Astronomical Society of the Pacific*, **26**, 258, 1914.

[5] *Astrophysical Journal*, **44**, 120, 1916.

given show clearly that the typical eclipsing binary of large range consists of a small, bright, dense component of the first spectral type (Class A or near it) and a large, faint component of much lower density and of the second type (Class G or redder).

It can hardly be doubted that, in such pairs, the component of low density represents the earlier stage of evolution, and that the terms "early" and "late," as applied to the spectral classes near the beginning and end of the series of the Draper classification, have no further significance than as describing the position of the stars in this sequence, and not at all in the sequence of evolution.

3. It is of much interest to compare the differences of color-index between the components with the differences of surface brightness (expressed likewise in stellar magnitudes), which may readily be computed from the data of Table XVIII. If the stars radiated according to Wien's law and were observed with monochromatic light, these two quantities should be in a fixed ratio to one another, no matter what the temperatures of the stars compared might be—namely, the ratio of the difference of the wave-lengths used in the visual and photographic observations to the photographic wave-length.

For bodies radiating according to Planck's law, this ratio would fall off slowly at very high temperatures. More serious deviations may arise from the departure of the stars from the laws of "black-body" radiation and from the fact that a considerable range of wave-lengths is effective in both the visual and the photographic observations. Nevertheless it might be expected that the difference in color-index would prove to be approximately proportional, at least, to that in surface brightness.

Table XIX contains all the data at present available which bear upon this question. The first column gives the name of the star; the second, the difference in color-index (ΔI) between the components, except for the last two stars, where the tabulated quantity is the difference in color-index between the light of the brighter component and that of the system at minimum, which is a mixture of the light of the two components (the eclipses being large, but partial). For RR Draconis and TW Andromedae two sets of values are given—one derived by comparison of the photo-visual

and photographic results of Seares[1] and Shapley,[2] respectively; and the other, from the same photographic observations and the visual measures of Shapley[3] and Van Biesbroeck.[4]

The third and fourth columns give the difference in surface brightness (ΔJ) between the components, expressed in stellar magnitudes and derived from the known depths of eclipse, and the ratios of the radii of the components. The solutions assuming the star-disks to be of uniform brightness, or darkened toward the limb, usually give different values of the ratio of the radii, and hence of the relative surface brightness, and both sets of values are therefore tabulated.

TABLE XIX

Star	Difference of Color-Index ΔI	Difference of Surface Brightness ΔJ		$\Delta I/\Delta J$	
		Uniform	Darkened	Uniform	Darkened
RR Draconis:					
Visual	$+0^{M}.84$	$+4^{M}.9$	$+4^{M}.1$	0.17	0.20
Photo-visual	.57	5.2	4.4	.11	.13
RW Tauri	.45	4.1	3.9	.11	.12
SW Cygni	.96	4.1	3.5	.23	.27
S Cancri	.76	4.1	3.4	.19	.22
W Delphini	.66	4.0	3.4	.16	.19
U Sagittae	.61	3.2	3.1	.19	.20
TW Andromedae:					
Visual	.33	3.0	2.4	.11	.14
Photo-visual	.51	2.9	2.2	0.18	0.23
RX Herculis	− .02	+0.14	+0.14	.....	.....
W Ursae Majoris	.00	0.00	0.00	.....	.....
Y Piscium	+ .37	>4.0	>3.5	.....	.....
RZ Cassiopeiae	+0.29	+3.6	+4.8	.....	.....

The elements of RZ Cassiopeiae are from Dugan's discussion,[5] the tabular quantities being the difference in surface brightness between those sides of the components which are visible at primary minimum. The geometrical elements of TW Andromedae are by Stewart.[6] The darkened elements of RW Tauri ($r_b=0.192$, $r_f=0.239$, $i=90°.0$) and U Sagittae ($r_b=0.226$, $r_f=0.282$, $i=90°.0$)

[1] *Astrophysical Journal*, **36**, 368, 1912.

[2] *Publications of the Astronomical Society of the Pacific*, **26**, 156, 1914.

[3] *Astrophysical Journal*, **37**, 154, 1913.

[4] *Annales de l'Observatoire Royal de Belgique*, **13**, 11, 1914.

[5] *Astrophysical Journal*, **44**, 117, 1916. [6] *Ibid.*, **42**, 315, 1915.

have been calculated from the new visual normals given above. The elements of the other stars may be found in Table XVIII or in Shapley's list[1] except in the case of RX Herculis, where the tabular data depend on Shapley's statement[2] that the visual surface brightness of the star eclipsed at principal minimum is 1.14 times that of the other, and Baker's value[3] 1.12 for the same ratio, as derived from his photographic observations.

The fifth and sixth columns give the ratios $\Delta I/\Delta J$, derived from the "uniform" and "darkened" solutions, for those cases in which the differences are large enough to permit a reliable determination. The results for different systems differ considerably, but not much more than those obtained by different observers for the same system. The mean values of $\Delta I/\Delta J$, giving each star equal weight, are 0.17 ± 0.010 for the uniform elements and 0.19 ± 0.012 for the "darkened"—the probable errors being derived in the usual way from the discordances between the individual values and the mean.

If the mean effective visual wave-length—to be exact, the "Crova wave-length"—is estimated as $0.52\,\mu$ for the visual and $0.42\,\mu$ for the photographic observations, the theoretical value of the ratio $\Delta I/\Delta J$ comes out 0.24; but neither of these wave-lengths has yet been determined even roughly for such methods of observation as are used in stellar photometry.

All that can be said at present is that, if, as seems probable, the color-indices given in this paper need to be increased considerably to reduce them to the standard scale, the observed ratio is in good accordance with theory.

For the two systems—RX Herculis and W Ursae Majoris—in which the components are nearly equal in surface brightness, they are also sensibly of the same color, as might be expected. For Y Piscium, where it is probable that the light at minimum is a mixture of that of the fainter component and of the uneclipsed portion of the brighter, the color-index is relatively small, as might again be anticipated.

[1] *Contributions from the Princeton University Observatory*, No. 3, pp. 82–91, 1915.

[2] *Astrophysical Journal*, **40**, 399, 1914.

[3] *Laws Observatory Bulletin*, **2**, 170, 1916.

The case of RZ Cassiopeiae is more interesting. According to Dugan's "uniform" elements, which are determined with great accuracy by a long series of observations covering the whole curve, the fainter component conceals 0.806 of the area of the brighter one at mid-eclipse, and the radius of the brighter star is 0.768 that of the fainter.

The depth of the principal minimum (measured from the adjacent portion of the light-curve, to eliminate the effects of ellipticity and mutual radiation) is $1^{M}.54$ visually and $1^{M}.77$ photographically, so that the ratio of the brightness at mid-eclipse to that outside eclipse is 0.242 in the first case and 0.196 in the second. If b and f denote the light of the two stars, we then have the equations:

$$\begin{aligned} f+0.194b &= 0.242(b+f) \quad \text{(visual)} \\ &= 0.196(b+f) \quad \text{(photographic)} \end{aligned}$$

whence $f/b=0.063$ visually and 0.002 photographically.

The first of these two results is in accordance with Dugan's data, it being remembered that the fainter side of the larger star is visible at this time; but the second appears to lead to the absurd conclusion that the companion gives out no violet light at all (within the errors of observation). A clue to the riddle may be found in the assumption that the brighter component is darkened toward the limb for the photographic wave-lengths, so that the light of the remaining uneclipsed portion is redder than that of the whole disk, leaving less to be accounted for by the redness of the companion. If the loss of brightness at the limb is x times the central brightness, the amount of light lost when the fraction a_0 of the apparent area of the disk is concealed by a star of $1/k$ times the radius of the one eclipsed is[1]

$$a_0(1-xX[k,a_0])$$

From the table of the function X it appears that in the present case

$$a_0=0.806,\ k=0.768,\ X=+0.071$$

and the fraction of the light of the brighter star which is lost by eclipse is $0.806+0.057x$. The equation for the photographic minimum then becomes

$$f+(0.194-0.057x)b=0.196(b+f)$$

whence

$$f/b=+0.002+0.071x.$$

[1] *Astrophysical Journal*, **36**, 402–403, 1912.

To the visual ratio $f/b=0.063$ corresponds a difference of surface brightness of $3^{M}.6$, which would lead us to anticipate a color-index of about $+0^{M}.6$ for the light of the fainter star relative to the brighter one and hence a photographic ratio $f/b=0.036$. This value may be exactly reproduced by setting $x=0.48$ in the equation derived above. It appears, therefore, that, if the brighter component of RZ Cassiopeiae is redder at the limb than at the center to the degree represented by the loss of about half the photographic light at the limb, the large faint comparison is of normal color-index in comparison with those of the other stars studied here. It is probable that this assumption is not far from the actual facts.

SUMMARY

1. Two thousand one hundred and one observations of the photographic brightness of six eclipsing variable stars have been made at Harvard and discussed at Princeton, and the resulting light-curves compared with those derived from visual observations.

2. In all cases it has been found possible to obtain a satisfactory representation of both the visual and the photographic observations by means of identical geometrical elements of the eclipsing system. The relative brightness of the two stars and in some cases the degree of darkening toward the limb are different for the visual and photographic rays.

3. The probable error of an estimate of magnitude depending on one Harvard plate is $\pm 0^{M}.068$, for those plates investigated in 1914–16. (For the older estimates in 1903 it is considerably higher.) The Harvard Photographic Library therefore contains material for very precise light-curves of all regular variable stars brighter than the twelfth magnitude.

4. The epochs of the middle of eclipse differ for the visual and photographic observations by amounts which, though small (twelve minutes at most), are much greater than can be explained by the accidental errors of observation. No explanation of these differences has been found.

5. In three cases—S Cancri, W Ursae Majoris, and RZ Cassiopeiae—there is definite evidence that the disks of the brighter components appear redder at the edge than at the center, as is the case

with the sun. In other cases there is little evidence of this effect, but it would be premature to say definitely that it was absent.

6. When the two components of an eclipsing system are nearly equal in surface brightness, they are of very nearly the same color, but when they differ considerably in surface brightness, the star of lower surface intensity is the redder. In many cases the difference in color-index is at least as great as that between the spectral classes A and G. The whiter components are usually smaller and undoubtedly denser than their reddish companions.

7. The difference of color-index is, on the average, about one-sixth of the difference of surface brightness, expressed in stellar magnitudes. It is, however, probable that the ratio will be increased to one-fifth or farther when the color-indices are referred to the standard system. When this ratio is definitely established, the way will be clear to estimate the linear diameters of all stars of known color-index and parallax, and the angular diameters of all stars of known spectral type.

8. The diagrams of the visual and photographic light-curves need no further explanation than to say (*a*) that, in order to avoid overlapping, the photographic curves have been displaced in a vertical direction relative to the visual curves, so that the photographic magnitudes are to be read from the scale at the right and the visual magnitudes at the left; and (*b*) that the phases used in plotting the normal points have in all instances been referred to the finally adopted instants of mid-eclipse, which differ, for the visual and photographic observations of the same star, by the quantities given in Table XVII, so that the differences between the times of photographic and visual minimum are not shown in the figures.

PRINCETON UNIVERSITY OBSERVATORY
February 22, 1917

Minor Contributions and Notes

ON THE AVERAGE PARALLAX OF THE STARS OF THE FOURTH TYPE

In the *Astrophysical Journal*, **32**, 9, 1910, Professor Kapteyn has published a determination of the average parallax of the stars of the fourth type. As this investigation has an important bearing on the question of the connection between the stars of type IV and Miss Maury's collateral series, I should like to point out that no reliable conclusion as to the mean parallax of these stars can be deduced from the data used by Kapteyn.

The material consists of the A.G. positions, some older observations, and the careful determinations made by Dr. Nörlund with the old meridian circle of the Copenhagen Observatory. Of this instrument Schjellerup says in the introduction to his star catalogue that the errors of the Pistor and Martin division amount in one case, at least, to 6″, and that one of the circles through an accident during transportation had become so deformed that the divisions were only indistinctly seen in the microscopes. Consequently the instrument has only been used in cases in which rough positions were better than none. In the present case the reliability of the positions has been further reduced by the method of observation; the bright stars were bisected by the wire, while the fainter ones were placed between two wires 14″ apart, so that the declinations are, strictly speaking, estimates rather than measures.

Although the systematic errors in the Copenhagen positions are partially reduced by the use of more catalogues in the computation of the proper motions, their existence is clearly revealed by a glance at the results. Arranging the secular motions μ_α and μ_δ according to right ascension, we get the means given in Table I.

It thus appears in Table II that the proper motions are positive from 0^h to 8^h and negative from 8^h to 24^h.

If star No. 60 in the group from 6^h to 8^h be rejected, the mean values become $+0^s.049$ and $-0''.68$, respectively.

From Tables I and II it appears that the proper motions in right ascension, if real, show a maximum in 7^h, even after the rejection stated, and a minimum in 19^h. This is clearly an effect of systematic errors in the proper motions, as it would require a right ascension of 13^h for the solar apex.

TABLE I

			No. of Stars
0^h to 2^h.....	$-0^s.118$	$-0''.13$	6
2 to 4	+0.083	+0.13	12
4 to 6	+0.012	−0.78	25
6 to 8	+0.149	−0.27	23
8 to 10	−0.049	+0.14	9
10 to 12	−0.070	+0.04	5
12 to 14	−0.047	−1.02	6
14 to 18	+0.172	+0.06	5
18 to 20	−0.028	+0.06	23
20 to 22	+0.020	−1.87	15
22 to 24	−0.060	−0.94	8

TABLE II

			No. of Stars
0^h to 4^h.....	$+0^s.016$	$+0''.04$	18
4 to 6	+0.012	−0.78	25
6 to 8	+0.149	−0.27	33
8 to 18	−0.009	−0.17	25
18 to 20	−0.028	+0.06	23
20 to 24	−0.008	−1.55	23

Arranging the proper motions according to declination, we get the synopsis in Table III.

In the neighborhood of the zenith of Copenhagen there is a strongly marked discontinuity in the proper motions in the right ascension from $+0^s.133$ to $-0^s.133$, the origin of which must be sought in Copenhagen rather than in the Milky Way. The change of sign has probably some connection with the change of position of the observer, and north of the zenith of Copenhagen the systematic errors of the method used in the observations of declination

reappear as a "star-drift" of $-2''$ common to seven stars! If the declinations of the stars between $+65°$ and $+70°$ be computed from the older observations alone, we find that the Copenhagen declinations in this region require a correction of about 1."

TABLE III

			No. of Stars
$-25°$ to $-15°$	$-0''.037$	$+0''.09$	10
-15 to -5	$+0.037$	-0.21	17
-5 to $+5$	-0.029	-0.97	15
$+5$ to $+15$	0.000	$+0.18$	11
$+15$ to $+25$	-0.055	-0.79	13
$+25$ to $+35$	$+0.027$	-0.28	13
$+35$ to $+45$	$+0.011$	-0.84	21
$+45$ to $+55$	$+0.133$	-0.75	14
$+55$ to $+65$	$+0.010$	-0.07	10
$+65$ to $+75$	-0.133	-2.17	7

The imaginary character of the proper motions is further clearly revealed by their rapid increase with the magnitude of the stars. Dividing the stars in groups of twenty, we obtain the mean values given in Table IV.

TABLE IV

$7^{M}58$	$2''.24$
8.26	2.86
8.80	2.61
9.01	4.31
9.31	5.86

As the systematic errors in the proper motions are of the same magnitude as the proper motions of the stars of the seventh magnitude, we may safely conclude that the secular motions found for the fainter ones are quite imaginary; hence any attempt at a determination of the mean parallax from these data must be regarded as mere guesswork.

H. E. Laŭ

Hörsholm
March 1917

RECENT CHANGES IN THE VARIABLE NEBULA N.G.C. 2261

This is a bright, comet-shaped nebula situated at R.A. 6^h34^m, Dec. $+8°49'$ (1900). Plates taken by the writer in the winter of 1915–1916 with the 24-inch reflector of the Yerkes Observatory, when compared under the blink-comparator with a plate taken with the same instrument by Mr. F. C. Jordan in March 1908, showed changes in the outline, and displacement of the structural details of the nebula. An account of the matter is given in this *Journal* for October 1916 (**44**, 190). The changes, though striking, were small, amounting at most to displacements of some 4 or 5 seconds of arc in the interval.

A plate taken by Mr. H. D. Curtis with the Crossley reflector of the Lick Observatory in January 1913 was very kindly lent to us by Director Campbell. This showed that the entire change had occurred between 1908 and 1913.

The nebula apparently remained quiescent until May 1916, when the last plate of that season was taken. In September of last autumn Miss Gushee of the Yerkes Observatory secured a plate of the nebula on which some changes were suspected. These were verified by a series of exposures which she made in the latter part of October.

A Lowell Observatory circular, dated January 29, 1917, stated that a comparison of plates taken, the one in March 1916, the other on January 25, 1917, by Mr. Lampland showed striking differences—apparent displacements which might amount to 15 seconds of arc.

On March 27, 1917, the writer secured a new plate with the Yerkes 24-inch reflector. In the telescope the nucleus was distinctly brighter than in the previous year, at least with respect to the neighboring nebulosity. The developed plate confirmed this impression to a remarkable degree. The general trend of the change seems to be that the nebula has faded very considerably all around its southern perimeter. It is to be regretted that the scale of the reflector is too small to bring out the finer details

distinctly. Plate IX has been prepared, showing the nebula as it appeared on February 27, 1916 and on March 27, 1917.

Although the interval is here an entire year, almost the whole change occurred after November 1 and had reached enormous proportions certainly by January 25, when a plate was taken at Lowell Observatory. It remains to be seen whether the change is still in progress.

The details of the change are confused by the small scale of the plate, as the image of the nebula is only half a millimeter in length on the negative. Clearly the former bright band just north of the nucleus has contracted to about a third of its length, and the whole southern portion of the body has contracted about the north-and-south axis—all this, however, without affecting the details in the outer portion, away from the nucleus. It would seem that the former illumination near the nucleus has faded away. The faint streamer running out from the nucleus in the direction south-following has perceptibly brightened. The recent plates have certain points in common with a plate taken by the late Dr. Isaac Roberts at Starfield on January 27, 1901, copies of which Mme Roberts had the kindness to prepare and send at the request of Director Frost.

The nucleus has long been known as the variable star R Monocerotis, for which a range between magnitudes 9.5 and 13.0 has been reported. The period is irregular. The photographs showed no appreciable variation until this spring, when the nucleus brightened by half a magnitude.

Director Slipher of the Lowell Observatory reports a continuous spectrum crossed by bright lines or bands, not those typical of the gaseous nebulae.

This nebula is of the same class as N.G.C. 6729, the variability of which was studied photographically by Knox-Shaw at Helwan. N.G.C. 2261, however, is larger and brighter, and hence the more easily studied in detail. Hind's variable nebula by T Tauri (N.G.C. 1555) is probably a third member of this class.

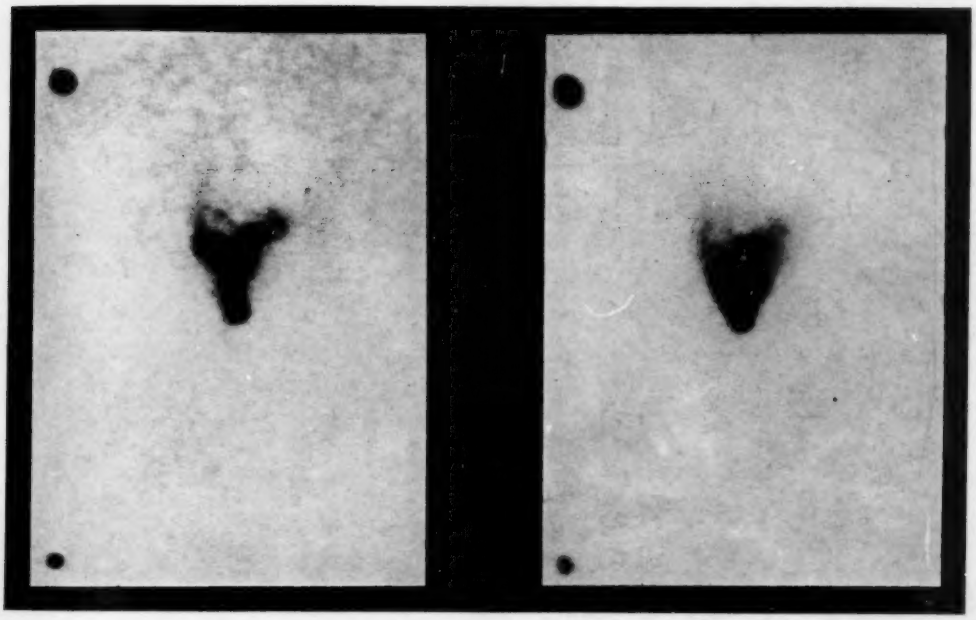
No attempt is here made to explain the phenomenon, save to remark that, unless we are witnessing a phenomenon of illumination, the nebula must be very near. Parallax and proper motion

PLATE IX

Scale: 1 mm = 3".6

North

E



W

1917 March 27 1916 February 27

TAKEN WITH YERKES 24-INCH REFLECTOR
Enlarged 24 times

N.G.C. 2261

are related to linear velocity across the line of sight by the formula $\pi=\frac{5\mu}{V}$. In this case μ could not be less than 20–30 seconds of arc, hence a velocity of, say, 500 km per second would point to a parallax of 0".2 at the least, and the matter could be settled by a direct measure of the parallax. If it is a question of illumination, then, as was pointed out by Slipher and Lampland, the nebula would not be more than ten or twelve thousand light-years distant at the most.

EDWIN P. HUBBLE

UNIVERSITY OF CHICAGO
April 10, 1917

Reviews

X-Rays. By G. W. C. Kaye. Second Edition. London and New York: Longmans, Green, & Co., 1917. Pp. xxi+285. $3.00 net.

"In this edition the writer, so far as his military duties would permit, has subjected the book to a thorough revision and incorporated original work of note published up to the middle of 1916." Although the first edition was far from outgrown, the rapid growth of this fascinating and highly important subject warrants a revision at this time. The additions which have been made are characterized by the same conciseness, yet comprehensiveness, found in the first edition. The substance of many scattered and often lengthy papers is clearly stated in remarkably few words.

Thirty pages in all have been added, twenty of which comprise a wholly new chapter on "X-Ray Equipment and Technique," which gives a brief account of the installation and procedure for taking X-ray photographs. From the standpoint of the physicist the changes of greatest interest are: (1) a careful revision of the chapter on "The Diffraction of X-Rays by Crystals," the recent work of the Braggs, De Broglie, Siegbahn, and others being incorporated, and the discoveries of Moseley being dealt with more at length; (2) a detailed description of the Coolidge tube; (3) a brief summary of the recent work of Barkla on the scattering of X-rays, on the gaseous absorption of X-rays, and on the velocity of the corpuscles ejected by X-rays; (4) a brief discussion of the quantum-theory relations which appear in X-ray phenomena.

Other changes and additions are: (1) the addition of sections on "The Ionics of an X-Ray Tube," "The Snook Hydrogen Tube," "Metal X-Ray Bulbs," "Energy Measurements," "Present Day Radiology and Radiotherapy," and "Recommendations for the Protection of X-Ray Workers"; (2) twenty-three new cuts have been added, one left out, and four changed; (3) three new tables have been added.

An undue prominence is given to the paper of Rutherford, Barnes, and Richardson (*Philosophical Magazine*, **30**, 339, 1915), the fundamental assumption of which (namely, that wave-lengths may be deduced from the absorption coefficients) has been shown to be incorrect, at any

rate for high frequencies, by several observers (see Duane and Hunt, *Physical Review*, **6**, 166, 1915; Hull, *ibid.*, **7**, 156, 1916; Allen and Alexander, *ibid.*, **9**, 198, 1917). As a result an entirely wrong impression is given in several places. On pp. 129 and 46 the statement is made that the shortest wave-length emitted by a Coolidge tube is 0.171×10^{-8} cm with a penetration of about one-half that of the gamma rays from radium C, and that this radiation appears at about 140,000 volts. Dr. Hull, of the General Electric Laboratory, has observed a radiation with wave-length 0.08×10^{-8} cm at a voltage of about 150,000, and he states that there is no indication of the existence of a limit to the hardness of the X-rays which may be generated by any particular anticathode (see *General Electric Review*, **19**, 314, 1916; also *Proceedings of the National Academy*, **2**, 265, 1916). On p. 230 credit is given to Rutherford for pointing out that absorption results are not a reliable guide for deducing wave-lengths. This was first pointed out by Duane and Hunt (*Physical Review*, **6**, 166, 1915) and has been verified by Hull and Rice (*Physical Review*, **7**, 326, 1916). As has been said, Rutherford worked on the opposite assumption. Siegbahn's discovery of an M series for the heavy metals (gold to thorium) softer than the K or L series (*Comptes Rendus*, **162**, 787,1 916) seems to have been overlooked (see p. 122). Moreover, I believe that the work of Webster (*Physical Review*, **6**, 599, 1916) shows clearly that the quantum relation $Ve = h\nu$, holds as well for the characteristic radiations as for the independent (the statement on p. 245 to the contrary), except that in this case the frequency to be used is that of the shortest (γ) line of the series.

The paper upon which the book is printed is not as good as that of the old edition, and as a consequence some of the cuts are a little blurred and not as clear as in the first edition. This was probably occasioned by war conditions and was unavoidable.

E. C. WATSON

RYERSON PHYSICAL LABORATORY

Annuaire pour l'an 1917 publié par le Bureau des Longitudes. Paris: Gauthier Villars et Cie, 1916. Pp. 661. Fr. 1.50.

The *Annuaire* for 1917 is devoted principally to astronomical tables and tables of measure and of meteorology. According to the plan adopted in 1904, it should contain also tables on geography and statistics. These have been omitted, together with those giving physical and chemical information.

Besides the usual ephemerides and lists of double stars, parallaxes, etc., there are tables for finding the value of gravity in different places, and for the calculation of altitude from barometric observations.

Revisions occur in the chapter on constellations, which contains six excellent charts with "catch-figures" outlined in red, and notes on the study of constellations, and in the chapter dealing with stellar spectra, where particular attention is given to recent developments. We find additions in the part dealing with the Julian and Gregorian calendars, and in the section on the rising and setting of the sun and moon.

Among the notices is one of particular interest to the physicist—"La Détermination du mètre en longueurs d'ondes lumineuses" by Maurice Hamy.

E. W. W.

Annuaire astronomique et météorologique pour 1917. Par Camille Flammarion. Paris: Ernest Flammarion. Pp. 436. 140 figures, charts, and diagrams. Fr. 2.

This well-known publication has this year the distinction of being the fifty-third issue by the same editor, M. Flammarion. In addition to the usual astronomical and meteorological tables, notices on the recent progress of astronomy, and a summary of celestial phenomena for the year, it contains also guides for the study of the constellations, descriptions of the sun, moon, planets, and stars, and methods of observation. Of particular interest this year are the references to the war situation in Europe and records of celestial observations in letters received from the front.

The book is well adapted to furnish inspiration for the amateur observer and general reader as well as to meet some needs of the more experienced astronomer.

V. M. G.

Astronomischer Jahresbericht. Bearbeitet im Kgl. Astronomischen Rechen-Institut. Berlin: Georg Reimer, 1914–1916. Bd. XV (Literatur des Jahres 1913), pp. 545, Marks 18; Bd. XVI, pp. 391, M. 14; Bd. XVII, pp. 299. M. ?

We are glad that this valuable annual of the literature of astronomy has been maintained even during war times, through the editorial efforts of Professor P. V. Neugebauer and the chief of the Rechen-Institut, Professor Fritz Cohn. The first volume of the three here considered

has its preface dated in June 1914. The subsequent shrinkage in size and in number of reviews is natural and is chiefly due, in the opinion of the editors, to the diminished output of scientific papers in war-ridden Europe.

Many papers not available to the editors were reviewed by colleagues at the observatory at Copenhagen. The shortening of the notices is not wholly a disadvantage. They are impersonal and generally adequate. The necessity of co-operation in the various phases of astronomy justifies the hope of a resumption of such relations between all the nations after the war.

F.

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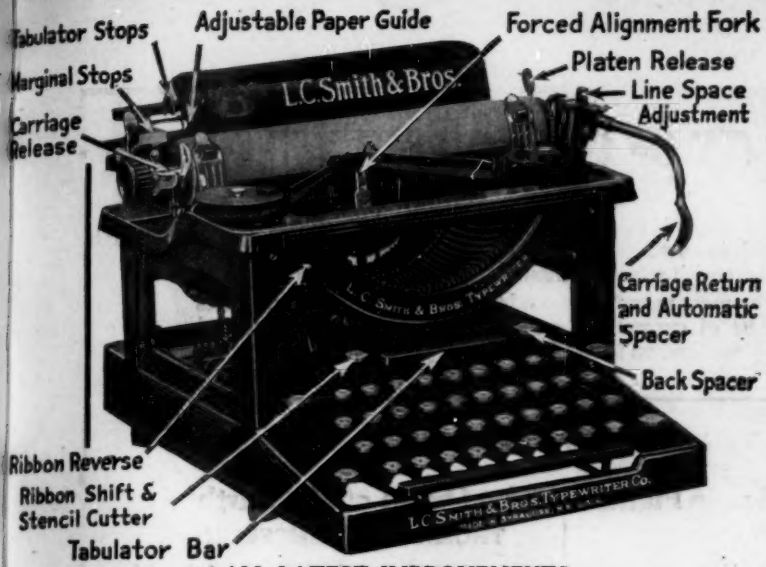

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